PTOLEMY

DAY 6

THE SEXAGESIMAL SYSTEM AND THE NEED FOR A TABLE OF CHORDS AND ARCS

Before getting into any of the detailed versions of Ptolemy's models for the motions of the Sun and the planets, we need to understand some of his mathematical equipment. To begin with, we need to understand a little bit about his numerical system, and also his need for developing a "table of chords and arcs," which will enable him to find the sizes of lengths and sides in triangles after being given some of the sizes of the other lengths and sides. We will not bother learning how to multiply, divide, and find square roots in Ptolemy's sexagesimal system. That is too much work with too little return, given what we have to do in the course. But it is important to understand what the sexagesimal system is, in order to understand Ptolemy's numbers, and it is useful to be able to turn his numbers into decimal form and our decimal expressions into sexagesimal ones.

DECIMAL SYSTEM.

Ptolemy does not use the decimal system to which we are accustomed, but the much more tedious sexagesimal system. Copernicus and Kepler and Newton also use it to some extent. And it remains in use today, in many applications.

What do we need to know about this system?

(a) It is important to know how to read sexagesimal values, and to be able to translate a sexagesimal value to a decimal one and vice versa, in order to understand Ptolemy's tables and calculations, at least as he presents them.

(b) It is good to know how to add and subtract them (which is just a matter of knowing how to simplify, and how to borrow).

(c) It is *not* important (in my humble opinion) to know how to find square roots in the sexagesimal system, or how to multiply or divide with them. This is tedious, uninteresting, and life is hard enough without it. Simply convert to decimal, do the calculation there, and convert back to sexagesimal.

We are accustomed to using the decimal system of numbers. This means not only that we have 10 basic numerical symbols (0, 1, 2, 3, 4, 5, 6, 7, 8, 9), but also that we signify numbers by making them sums of powers of ten. For example:

$$356 \quad = \quad (3 \times 10^2) + (5 \times 10^1) + (6 \times 10^0).$$

Notice that the place of the digit indicates which power of ten we are multiplying. And if we need to designate fractions of a unit, we do so by cutting it up into equal parts in numbers which are also powers of ten, and then the place of the digit again indicates which power of ten that digit is to be multiplied by:

$$.238 \quad = \quad (2 \times 10^{-1}) + (3 \times 10^{-2}) + (8 \times 10^{-3})$$

Or, putting it a bit differently,

"3.1415" means $3 + \frac{1}{10^1} + \frac{4}{10^2} + \frac{1}{10^3} + \frac{5}{10^4}$

and generally

"a.bcde ..." means $\frac{a}{1} + \frac{b}{10^1} + \frac{c}{10^2} + \frac{d}{10^3} + \frac{e}{10^4} \cdots$

SEXAGESIMAL SYSTEM.

This way of representing numbers uses 60 as a base instead of 10. Instead of saying how many tenths (or hundredths etc.) we have, we say how many sixtieths (or thirty-six-hundredths etc.) we have.

And usually we only take things to the second sexagesimal place after the whole number, i.e. to the 3600ths place. The whole number place is signified by a superscript of whatever unit we are using (e.g. H for Hours, or ° for Degrees) and the "firsts" place by one superscript minute-mark, and the "seconds" place by two superscript minute-marks, thus:

37^{H} 14' 53''

which would be read "37 Hours, 14 Minutes, 53 Seconds."

If we are dividing arcs of a circle, or angles, then we write:

37° 14' 53''

which we read "37 degrees, 14 arc-minutes, 53 arc-seconds."

We still divide time sexagesimally, i.e. we divide 1 hour not into "10 minutes" but into "60 minutes", and one minute not into "10 seconds" but into "60 seconds" (3600ths of an hour).

So generally

a b' c'' d''' ... means $\frac{a}{1} + \frac{b}{60^1} + \frac{c}{60^2} + \frac{d}{60^3} + \frac{e}{60^4} \cdots$

FROM SEXAGESIMAL TO DECIMAL

To convert from sexagesimal to decimal is easy:

$24^\circ\ 31'\ 12'' \quad = \quad 24 + 31/60 + 12/3600 \quad = \quad 24.52^\circ$

FROM DECIMAL TO SEXAGESIMAL

This is more painful, but still not too bad. Suppose you want to translate the decimal expression 35.398° into sexagesimal form.

$$35.398 = 35 + 398/1000$$

Now we want to turn that fraction into a fraction over 3600 (if we are going to take it out only to the "seconds" place). So we multiply 398 by 3.6 (since that is what we want to multiply the denominator by), giving us 1432.8, which we can round up to 1433. Now we have

$$35.398 = 35 + 1433/3600$$

But just as every 60 seconds is a minute, so too every 60 of our 3600ths is a "first" or "minute" in sexagesimal notation. So how many 60s are there in 1433?

$$1433 \div 60 = 23.88333...$$

So there are basically 23 sixties in there. But $23 \times 60 = 1380$, and

$$1433 - 1380 = 53$$

leaving us with

$$35.398 = 35 + 23/60 + 53/3600$$

i.e. $35.398 = 35^\circ\ 23'\ 53''$

EXERCISES

Turn the following decimal expressions into "hours, minutes, seconds" :

23.468 hours
5.203 hours
14.777 hours