

Chapter 13

Brahe and Kepler

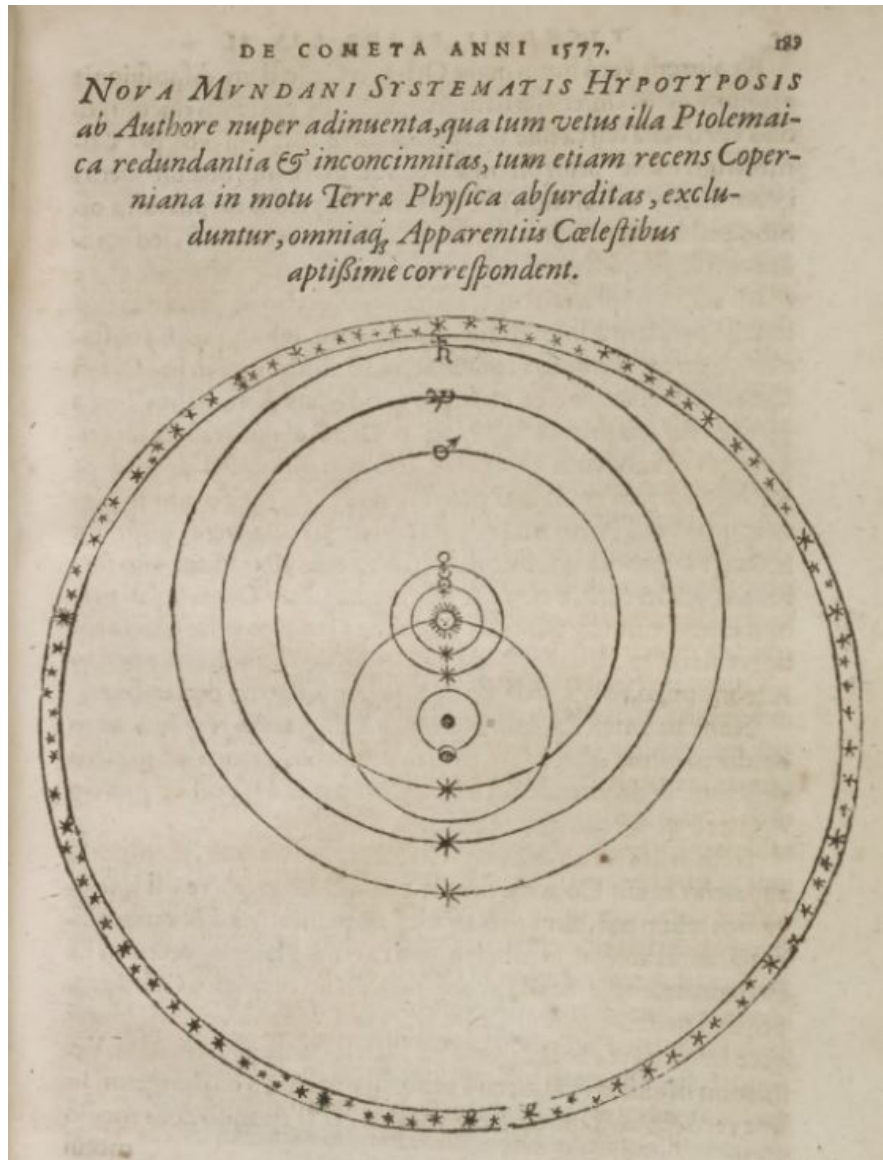
Tycho Brahe (1546-1601) was a Danish astronomer who made contributions to the development of astronomy in both the areas of observation and theory. He spent much of his career trying to make accurate observations of the heavens, and he constructed an observatory that contained several large instruments with which he could make observations to small fractions of degrees. His systematic and detailed observations of Mars, in particular, would be of crucial significance to Johannes Kepler. Brahe also observed a new star (a nova) that appeared in the sky in 1572. Because he could detect no parallax for this star (in other words, he could not see it move against the background of stars when viewed from different places on earth), he realized that this new object was not located in the upper atmosphere, but must be in the heavens. Likewise, he observed a comet that was visible in 1577-78, and using his own measurements and others', he was able to show that it was traveling through the heavens.

Partly because of these phenomena, he devised his own system of the world (depicted below). He thought that the earth is at rest at the center of the universe and that all of the heavenly bodies revolve around the earth every day; however, in his model the planets all simultaneously orbit not the earth, but the sun, which travels around the earth on its own orbit in a year. This system could be considered a hybrid between the models of Ptolemy and Copernicus, but, unlike either of these men, Brahe did not embed the planets in large, clear ethereal spheres; instead, he thought that the planets were hard bodies traveling through a fluid material.

Consider the advantages and disadvantages of this system over those of Ptolemy and Copernicus.

Why do you think his observations of a nova and comet were significant? How would a follower of Ptolemy react to these observations?

Consider how far away the stars must be in Copernicus's universe in order for us to not notice any parallax when we view the stars from opposite points on our orbit.



Johannes Kepler (1571-1630) was a talented mathematician and astronomer who worked under Brahe. He was tasked with using Brahe's detailed observations to make a model for Mars. After struggling with this project for years, he finally succeeded and his results were published in his *Astronomia Nova*. While his first two laws of planetary motion are contained in this book in a limited way (he only discusses Mars), his later work *Harmonies of the World* contains the complete versions of his famous three laws of planetary motion.

Kepler, *Astronomia Nova* [1609], Introduction:

My aim in the present work is chiefly to reform astronomical theory (especially of the motion of Mars) in all three forms of hypotheses [Ptolemaic, Copernican, and Brahean], so that our computations from the tables correspond to the celestial phenomena. Hitherto, it has not been possible to do this with sufficient certainty.

In fact, in August of 1608, Mars was a little less than four degrees beyond the position given by calculation from the Prutenic tables. In August and September of 1593, this error was a little less than five degrees, while in my new calculation the error is entirely suppressed.

Kepler, *Harmonies of the World* [1619], book V, chapter 3:

In the *Commentaries on Mars* [*Astronomia Nova*, 1609] I have demonstrated from the sure observations of Brahe that daily arcs, which are equal in one and the same eccentric circle, are not traversed with equal speed; but that these differing delays in equal parts of the eccentric observe the ratio of their distances from the sun, the source of movement; and conversely, that if equal times are assumed, namely, one natural day in both cases, the corresponding true diurnal arcs of one eccentric orbit have to one another the ratio which is the inverse of the ratio of the two distances from the sun.¹ Moreover, I demonstrated at the same time that the planetary orbit is elliptical and the sun, the source of movement, is at one of the foci of the ellipse.²

. . .

So far we have dealt with the different delays or arcs of one and the same planet. Now we must also deal with the comparison of the movements of two planets. . . . After finding the true intervals of the spheres by the observations of Tycho Brahe and continuous labour and much time, at last, at last the right ratio of the periodic times to the spheres

Though it was late, looked to the unskilled man,
Yet looked to him, and, after much time, came,

and, if you want the exact time, was conceived mentally on the 8th of March in this year One Thousand Six Hundred and Eighteen but unfelicitously submitted to calculation and rejected as false, finally, summoned back on the 15th of May, with a fresh assault undertaken, outfought the darkness of my mind by the great proof afforded by my labor of seventeen years on Brahe's observations and meditation upon it uniting in one concord, in such fashion that I first believed I was dreaming and was presupposing the object of my search among the principles. But it is absolutely certain and exact that the ratio which exists between the periodic times of any two planets is precisely the ratio of the 3/2th power of the mean distances, i.e., of the spheres themselves;³ provided, however, that the arithmetic

1. Dr. Zepeda's and Shields' note: This implies "Kepler's second law": that planets in their orbit, by a line drawn from them to the sun, sweep out equal areas in equal times. One can get a rough sense of this by thinking of a planet as sweeping out a triangle instead of a curved area. For the areas of triangles are proportional to the ratio compounded of the ratio of their bases and the ratio of their heights (in modern terms $\text{area} = \frac{1}{2}(\text{base} \times \text{height})$). The distance of the planet from the sun corresponds to the height of its swept out triangle, and the diurnal arc corresponds to the base of the swept out triangle. Compounding one ratio with its inverse ratio yields equality.

2. Dr. Zepeda's and Shields' note: This is "Kepler's first law."

3. Dr. Zepeda's and Shields' note: This is "Kepler's third law."

mean between both diameters of the elliptic orbit be slightly less than the longer diameter. And so if any one take the period, say, of the Earth, which is one year, and the period of Saturn, which is thirty years, and extract the cube roots of this ratio and then square the ensuing ratio by squaring the cube roots, he will have as his numerical products the most just ratio of the distances of the Earth and Saturn from the sun.⁴ For the cube root of 1 is 1, and the square of it is 1; and the cube root of 30 is greater than 3, and therefore the square of it is greater than 9. And Saturn, at its mean distance from the sun, is slightly higher than nine times the mean distance of the Earth from the sun.

4. For in the *Commentaries on Mars*, chapter 48, page 232, I have proved that this Arithmetic mean is either the diameter of the circle which is equal in length to the elliptic orbit, or else is very slightly less.