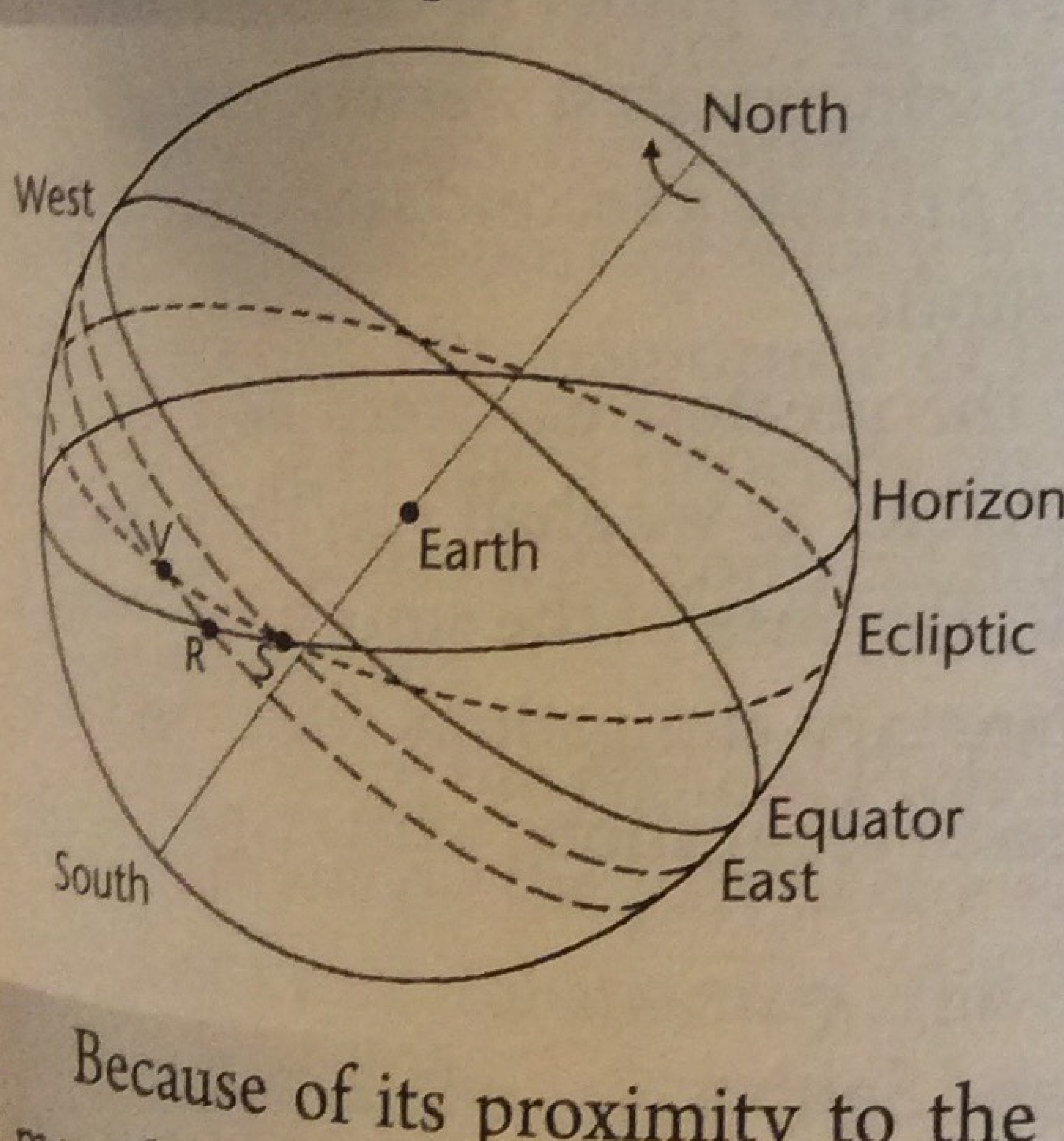
PRELIMINARIES TO BOOK IX

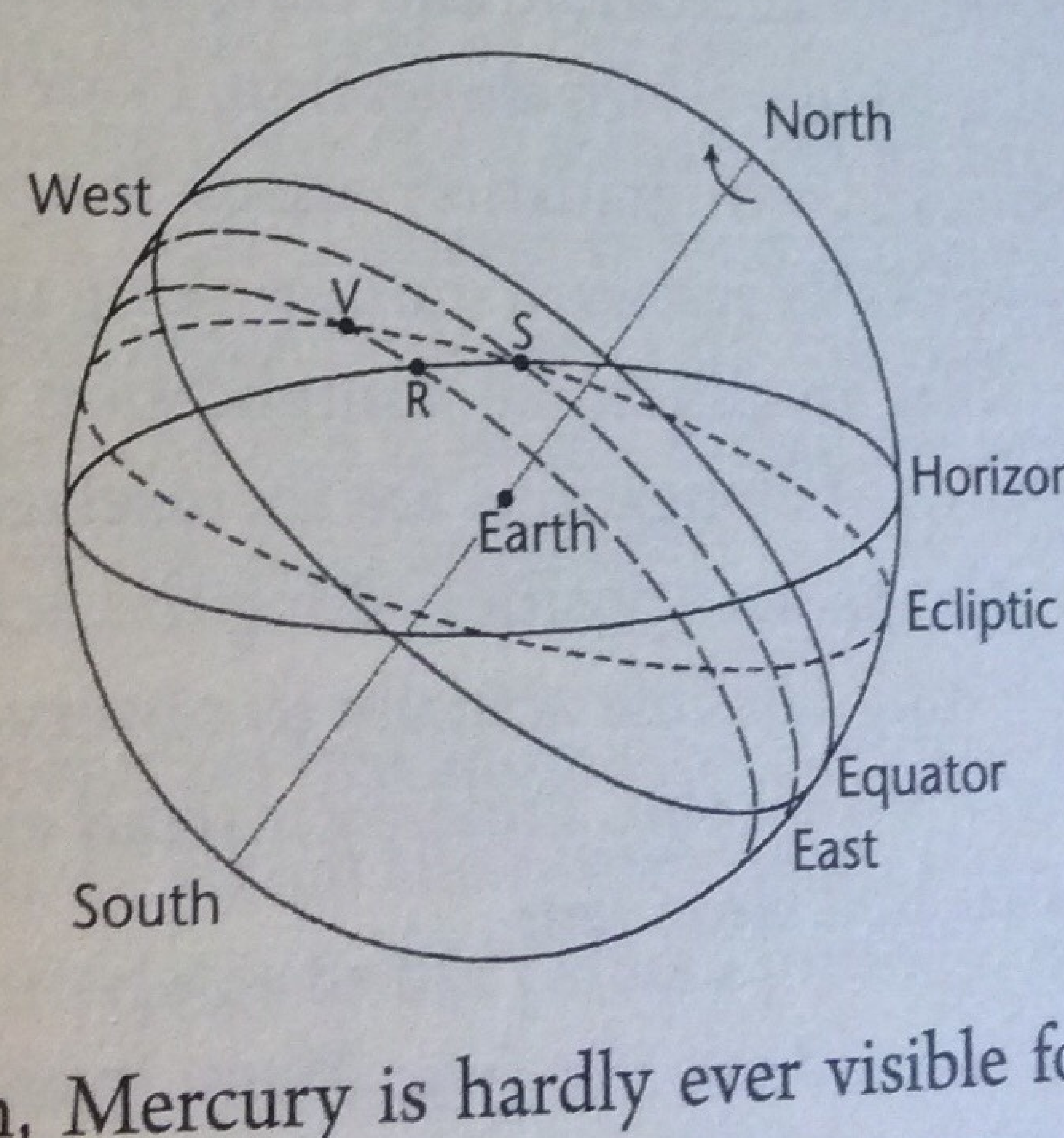
Mercury and Venus

1. *Appearance and motion in the heavens.* Mercury is ordinarily visible for only two two-week periods each year, near the horizon at sunrise or sunset. Venus too alternates between sunrise and sunset appearances, but it remains visible for longer periods than does Mercury and is far brighter, being surpassed in brilliance only by the sun and the moon.

Like the sun, Mercury and Venus move generally eastward through the zodiac. Each is always found within a characteristic distance from the sun—about 28 degrees for Mercury, 47 degrees for Venus. Each of them, therefore, completes one cycle about the zodiac in the same average time that the sun does.[1] Their apparent motions, however, are even more irregular than is the sun's. Venus, for example, may sometimes pass twice as quickly through one zodiac sign as it does through another.

Each of these planets appears sometimes as a morning star, rising in the east shortly before sunrise, and sometimes as an evening star, setting in the west shortly after sunset. The lefthand diagram shows Venus, *V*, as morning star; it rises at *R* before the sun rises at *S*. The righthand diagram depicts Venus as evening star; it sets at *R* after the sun sets at *S*.





Because of its proximity to the sun, Mercury is hardly ever visible for more than 60 or 70 minutes before sunrise or after sunset. Venus, by contrast, may sometimes be viewed as much as $3\frac{1}{2}$ hours before sunrise or after sunset.

---

[1] The basis for this inference is discussed in the section "The Planets and the Mean Sun," below in these Preliminaries.

It is not obvious by sight alone that a planet's morning and evening appearances are in fact manifestations of the same planet. The Greeks in Homer's time regarded Venus the morning star and Venus the evening star as two different planets, which they named *Phosphoros* and *Hesperos*, respectively. By the time of Plato, however, it was recognized that these two were actually one planet, periodically rendered invisible by the light of the sun.

2. *Elongation and retrogradation.* When either Mercury or Venus emerges as a morning star from a period of invisibility (its *apparition*) it is west of, and very close to, the sun—so that it rises in the morning just before the sun does; it is said to exhibit a *western elongation,* measured in degrees along the ecliptic. In subsequent days its western elongation grows larger and larger until it reaches a maximum—the *greatest western elongation,* abbreviated GWE. The elongation then decreases until the planet becomes hidden by the sun's light. After a period of invisibility it reappears close to the sun, but now as an evening star, and therefore east of the sun along the ecliptic; it is said to exhibit an *eastern elongation.* In subsequent days its eastern elongation increases to a maximum—a *greatest eastern elongation* or GEE—after which the elongation again decreases until the planet is once more rendered invisible by the sun and the whole cycle begins anew.

At some time after reaching a GEE, but before attaining the following GWE, the planet appears to stop its eastward motion with respect to the ecliptic; it is then said to have reached a *station.* It next begins to move westward along the ecliptic; this western movement is called *retrogradation.* Retrogradation continues as the planet's eastern elongation decreases to the point of occultation, and continues further as the planet re-emerges with an increasing western elongation. Prior to attaining greatest western elongation, however, retrogradation ceases: the planet exhibits a second station and then resumes its eastward motion along the ecliptic.

It is an additional complication that the greatest eastward and greatest westward elongations are themselves variable; as we shall see, they depend on the planet's position along the ecliptic. Because of the sun's brightness, it is hardly possible actually to observe planetary elongations with respect to the apparent sun. Ptolemy instead expresses elongations with respect to the calculated *mean sun.*

Mars, Jupiter, and Saturn

3. *Appearance and motion in the heavens.* Mars is famously reddish in color. It varies considerably in brilliance, sometimes approaching the brightness of Jupiter, which in turn is brighter than any star other than Venus. Saturn shines about as brightly as do the brightest among the fixed stars.

Like the other planets, these three planets move generally eastward along the zodiac. On the average, Mars completes one cycle about the zodiac in about two years, Jupiter in about twelve years, and Saturn in about twenty-nine years. Nevertheless their apparent rates of progress can vary greatly. For example, Mars may move through a single zodiac sign in as little as one month or as long as several months.

4. *Elongation and retrogradation.* Unlike Mercury and Venus, the planets Mars, Jupiter, and Saturn can appear at any angular distance from the sun; they never exhibit a greatest elongation. Thus at any time of the year each of them can be anywhere on the ecliptic. When these planets are near the sun they are said to be in *conjunction*, and are then difficult or impossible to see. These periods of invisibility may last for several months.

When one of these planets is opposite the sun on the ecliptic it is said to be in *opposition*. It then rises when the sun is setting and sets when the sun is rising. When the planet is near opposition, it appears to cease its eastward motion through the zodiac; as with Mercury and Venus, this cessation of motion is called a *station*. Following this station, the planet reverses its motion and travels westward along the ecliptic; as before, this westward motion is termed *retrogradation*. The planet then reaches a second station, after which it resumes its eastward motion through the zodiac.

In the case of Mars, the appearance of station may last for as long as two weeks; retrogradation may last from about 40 to about 90 days, depending on where it is in the zodiac. All three planets, but especially Mars, appear noticeably brighter when they are in retrograde motion.

Planetary Anomalies

5. *Zodiacal and heliacal anomaly.* A celestial body's departure from uniform motion is termed an anomaly. In the case of the sun we observe only a single anomaly: the sun's position on the ecliptic sometimes exceeds, sometime falls short of where it would be if it always moved uniformly. Furthermore, the magnitude of this discrepancy depends on the time of year; that is to say, it depends on where the sun is in the zodiac. Because of its connection with zodiacal position, the sun's anomaly is considered a *zodiacal* anomaly.

In the case of the planets, Ptolemy identifies two anomalies. One, like the sun's, depends on the part of the zodiac in which the planet appears and is therefore a zodiacal anomaly. The other relates to the planet's position, not with respect to the zodiac but with respect to the sun; it is therefore called a *heliacal* anomaly. As Ptolemy acknowledges in Chapter 2 below, the two anomalies are intermixed and thus difficult to distinguish. The following tables present some modern data for Venus in a way that aims to separate that planet's two anomalies. Familiarity with these data, and the phenomena they record, will also prove useful for understanding Ptolemy's treatment of Venus in Book X.

Table 1. Positions and Elongations of Venus, Jan 2010–May 2012

Date	Venus	Mean Sun	Elongation
2010			
Jan 1	277°51′	280°32′	2°42′ W
Jan 6	284 08	285 28	1 20
Jan 11	290 26	290 24	0 02
Jan 16	296 43	295 19	1 24 E
Jan 21	303 00	300 15	2 45
Jan 26	309 17	305 11	4 06
Jan 31	315 34	310 06	5 27
Feb 5	321 50	315 02	6 48
Feb 10	328 06	319 58	8 09
Feb 15	334 22	324 54	9 29
Feb 20	340 38	329 49	10 48
Feb 25	346 53	334 45	12 08
Mar 2	353 07	339 41	13 26
Mar 7	359 21	344 36	14 44
Mar 12	5 34	349 32	16 02
Mar 17	11 46	354 28	17 19
Mar 22	17 58	359 23	18 35
Mar 27	24 09	4 19	19 50
Apr 1	30 20	9 15	21 05
Apr 6	36 29	14 10	22 18
Apr 11	42 37	19 06	23 31
Apr 16	48 45	24 02	24 43
Apr 21	54 52	28 58	25 54
Apr 26	60 57	33 53	27 04
May 1	67 02	38 49	28 13
May 6	73 05	43 45	29 20
May 11	79 07	48 40	30 27
May 16	85 09	53 36	31 32
May 21	91 08	58 32	32 37
May 26	97 07	63 27	33 39
May 31	103 04	68 23	34 40
Jun 5	108 59	73 19	35 40
Jun 10	114 52	78 15	36 38
Jun 15	120 44	83 10	37 34
Jun 20	126°34′	88°06′	38°28′ E
Jun 25	132 21	93 02	39 20
Jun 30	138 06	97 57	40 09
Jul 5	143 48	102 53	40 55
Jul 10	149 27	107 49	41 39
Jul 15	155 03	112 44	42 19
Jul 20	160 35	117 40	42 55
Jul 25	166 03	122 36	43 27
Jul 30	171 25	127 31	43 54
Aug 4	176 43	132 27	44 15
Aug 9	181 53	137 23	44 31
Aug 14	186 57	142 19	44 38
Aug 19	191 52	147 14	44 38
Aug 24	196 37	152 10	44 27
Aug 29	201 09	157 06	44 04
Sep 3	205 28	162 01	43 27
Sep 8	209 29	166 57	42 32
Sep 13	213 10	171 53	41 18
Sep 18	216 26	176 48	39 38
Sep 23	219 11	181 44	37 27
Sep 28	221 19	186 40	34 39
Oct 3	222 42	191 35	31 07
Oct 8	223 14	196 31	26 42
Oct 13	222 47	201 27	21 20
Oct 18	221 21	206 23	14 59
Oct 23	219 02	211 18	7 44
Oct 28	216 08	216 14	0 05 W
Nov 2	213 08	221 10	8 02
Nov 7	210 30	226 05	15 36
Nov 12	208 37	231 01	22 23
Nov 17	207 44	235 57	28 13
Nov 22	207 51	240 52	33 02
Nov 27	208 56	245 48	36 53
Dec 2	210 51	250 44	39 53
Dec 7	213 29	255 39	42 11

The table lists longitudes (degrees eastward along the zodiac) of Venus and the mean sun at five-day intervals over a 29-month period; values are rounded to the nearest minute. Notice the following events: GEE about Aug 17, 2010, station about Oct 8, 2010 and again about Nov 20, 2010, and GWE about Jan 6, 2011. Another GEE occurs about Mar 26, 2012.

Date	Venus	Mean Sun	Elongation
Dec 12	216°41′	260°35′	43°54′W
Dec 17	220 23	265 31	45 08
Dec 22	224 28	270 27	45 59
Dec 27	228 52	275 22	46 30
2011			
Jan 1	233 34	280 18	46 44
Jan 6	238 28	285 14	46 46
Jan 11	243 33	290 09	46 36
Jan 16	248 47	295 05	46 18
Jan 21	254 09	300 01	45 51
Jan 26	259 38	304 56	45 18
Jan 31	265 12	309 52	44 40
Feb 5	270 51	314 48	43 57
Feb 10	276 33	319 44	43 10
Feb 15	282 19	324 39	42 20
Feb 20	288 08	329 35	41 27
Feb 25	293 59	334 31	40 32
Mar 2	299 52	339 26	39 34
Mar 7	305 47	344 22	38 35
Mar 12	311 43	349 18	37 35
Mar 17	317 41	354 13	36 33
Mar 22	323 39	359 09	35 30
Mar 27	329 39	4 05	34 26
Apr 1	335 40	9 00	33 21
Apr 6	341 41	13 56	32 15
Apr 11	347 43	18 52	31 09
Apr 16	353 45	23 48	30 03
Apr 21	359 47	28 43	28 56
Apr 26	5 50	33 39	27 49
May 1	11 54	38 35	26 41
May 6	17 57	43 30	25 33
May 11	24 01	48 26	24 25
May 16	30 05	53 22	23 17
May 21	36 09	58 17	22 08
May 26	42 14	63 13	20 59
May 31	48 19	68 09	19 50
Jun 5	54 24	73 04	18 41
Jun 10	60 29	78 00	17 31
Jun 15	66 34	82 56	16 21

Date	Venus	Mean Sun	Elongation
Jun 20	72°40′	87°52′	15°11′W
Jun 25	78 46	92 47	14 01
Jun 30	84 53	97 43	12 50
Jul 5	91 00	102 39	11 39
Jul 10	97 07	107 34	10 27
Jul 15	103 15	112 30	9 15
Jul 20	109 23	117 26	8 02
Jul 25	115 32	122 21	6 49
Jul 30	121 41	127 17	5 36
Aug 4	127 51	132 13	4 22
Aug 9	134 01	137 09	3 07
Aug 14	140 11	142 04	1 53
Aug 19	146 23	147 00	0 37
Aug 24	152 34	151 56	0 38 E
Aug 29	158 46	156 51	1 54
Sep 3	164 58	161 47	3 11
Sep 8	171 10	166 43	4 27
Sep 13	177 22	171 38	5 44
Sep 18	183 35	176 34	7 01
Sep 23	189 48	181 30	8 18
Sep 28	196 01	186 25	9 35
Oct 3	202 14	191 21	10 53
Oct 8	208 27	196 17	12 10
Oct 13	214 40	201 13	13 27
Oct 18	220 53	206 08	14 45
Oct 23	227 06	211 04	16 02
Oct 28	233 19	216 00	17 20
Nov 2	239 32	220 55	18 37
Nov 7	245 45	225 51	19 54
Nov 12	251 58	230 47	21 11
Nov 17	258 10	235 42	22 28
2011			
Nov 22	264 23	240 38	23 45
Nov 27	270 35	245 34	25 01
Dec 2	276 47	250 29	26 18
Dec 7	282 59	255 25	27 34
Dec 12	289 10	260 21	28 49
Dec 17	295 21	265 17	30 04
Dec 22	301 31	270 12	31 18

Date	Venus	Mean Sun	Elongation
Dec 27	307°40′	275°08′	32°32′E
2012			
Jan 1	313 49	280 04	33 45
Jan 6	319 57	284 59	34 57
Jan 11	326 03	289 55	36 08
Jan 16	332 07	294 51	37 17
Jan 21	338 11	299 46	38 25
Jan 26	344 13	304 42	39 31
Jan 31	350 12	309 38	40 34
Feb 5	356 09	314 34	41 36
Feb 10	2 03	319 29	42 34
Feb 15	7 54	324 25	43 29
Feb 20	13 41	329 21	44 21
Feb 25	19 25	334 16	45 09
Mar 1	25 04	339 12	45 52
Mar 6	30°37′	344°08′	46°29′E
Mar 11	36 04	349 03	47 01
Mar 16	41 25	353 59	47 26
Mar 21	46 38	358 55	47 43
Mar 26	51 41	3 50	47 51
Mar 31	56 34	8 46	47 48
Apr 5	61 15	13 42	47 33
Apr 10	65 40	18 38	47 02
Apr 15	69 47	23 33	46 14
Apr 20	73 34	28 29	45 05
Apr 25	76 55	33 25	43 30
Apr 30	79 44	38 20	41 24
May 5	81 56	43 16	38 40
May 10	83 24	48 12	35 12

6. *Station and retrogradation.* In the preceding table, notice Venus' stations on about Oct 8 and Nov 20, 2010 as well as its retrograde motion between those stations.

7. *Greatest, least, and mean notion.* As Ptolemy will note in Chapter 3 below, the motion of Venus (averaged over a sufficiently long time) is equal to the motion of the mean sun. On a daily basis, therefore, the planet's mean motion must be equal to the motion of the mean sun, about 0° 59′ per day.

Notice in the preceding table that when the planet's elongation is zero, it is either overtaking the mean sun as on Jan 11, 2010, or falling behind it as on Oct 28, 2010. Examining the changes in Venus's position over the intervals before and after these dates reveals that the planet exhibits its *greatest eastward motion* as it overtakes the mean sun and its *least eastward motion* (or, rather, its greatest retrogradation) as it falls behind. These episodes correspond to what we earlier identified as greatest and least motions, respectively, of the apparent sun.

Furthermore, an occasion of GEE or GWE is also an occasion of *mean motion*—since when the planet exhibits a greatest elongation it must have ceased increasing its distance from the mean sun, but is not yet decreasing it. If its distance from the mean sun is neither increasing nor decreasing, the planet must be moving at the same rate as the mean sun. But the motion of the mean sun is, as we have noted, equal to the mean motion of the planet.

Observe, then, in the preceding table, that the planet's time from greatest to mean motion is from Jan 11 to Aug 17, 2010 or 218 days; while its time from mean to least motion is from Aug 17 to Oct 28, 1010 or 72 days. Thus the planet's time from greatest to mean motion is greater than its time from mean to least motion. Ptolemy will draw a conclusion from this inequality in Chapter 5 below.

8. *Magnitude of greatest elongation.* In the preceding table, notice that the GEE of Aug 17, 2010 (43°38′) does not equal the GEE of Mar 26, 2012 (47°51′). In fact, a survey of greatest elongations attained over a sufficient range of years reveals that both GEE and GWE vary between minimum and maximum values. The following table summarizes one such survey.

Table 2. Successive GEE and GWE of Venus, 1900–1939

Date	Mean Sun	GEE	Date	Mean Sun	GWE
1900 Apr 27	34°31′	47°14′	1900 Sep 19	177°26′	47°52′
1901 Dec 10	258 01	46 25	1902 Apr 28	35 02	44 26
1903 Jul 5	102 03	45 23	1903 Nov 25	242 46	47 55
1905 Feb 18	327 17	48 02	1905 Jul 12	109 13	45 48
1906 Sep 19	176 59	44 33	1907 Feb 6	314 59	45 39
1908 Apr 25	32 37	47 17	1908 Sep 17	175 32	47 50
1909 Dec 7	255 08	46 19	1910 Apr 25	32 07	44 26
1911 Jul 3	99 54	45 27	1911 Nov 23	240 51	47 58
1913 Feb 16	325 23	48 01	1913 Jul 9	106 20	45 43
1914 Sep 16	174 06	44 32	1915 Feb 3	312 05	45 44
1916 Apr 22	29 43	47 21	1916 Sep 15	173 37	47 46
1917 Dec 5	253 13	46 15	1918 Apr 23	30 13	44 26
1919 Jul 1	98 00	45 31	1919 Nov 20	237 57	48 00
1921 Feb 13	322 29	48 00	1921 Jul 7	104 25	45 39
1922 Sep 14	172 11	44 32	1923 Feb 1	310 11	45 49
1924 Apr 20	27 48	47 24	1924 Sep 13	171 73	47 43
1925 Dec 2	250 19	46 09	1926 Apr 20	27 20	44 25
1927 Jun 28	95 06	45 34	1927 Nov 18	236 03	48 03
1929 Feb 11	320 34	47 58	1929 Jul 4	101 31	45 35
1930 Sep 11	169 17	44 31	1931 Jan 29	307 17	45 54
1932 Apr 18	25 54	47 27	1932 Sep 10	168 49	47 40
1933 Nov 30	248 25	46 04	1934 Apr 18	25 25	44 26
1935 Jun 26	93 11	45 38	1935 Nov 16	234 08	48 05
1937 Feb 9	318 40	47 57	1937 Jul 2	99 37	45 31
1938 Sep 8	166 24	44 32	1939 Jan 27	305 22	45 59

In the previous table, dates are rounded to the nearest day at 00:00 UT; angles are rounded to the nearest minute. Notice that in any eight-year period beginning with a GEE, Venus attains GEE again five times, ending up with a nearly equal value of GEE on almost the same month and day at which it began. The same is true for GWE.

The values recorded in the previous table indicate that the magnitudes of GEE and GWE depend upon the month and day on which each greatest elongation is attained—or, what is the same, that they depend on the position of the mean sun. That dependency becomes manifest when we tabulate the greatest elongations in the order of the mean sun's longitude:

Table 3. Venus GEE and GWE by Longitude of Mean Sun

Date	Mean Sun	GEE	GWE
1934 Apr 18	25°25′		44°26′
1932 Apr 18	25 54	47°27′	
1926 Apr 20	27 20		44 25
1924 Apr 20	27 48	47 24	
1916 Apr 22	29 43	47 21	
1918 Apr 23	30 13		44 26
1910 Apr 25	32 08		44 26
1908 Apr 25	32 37	47 17	
1900 Apr 27	34 31	47 14	
1902 Apr 28	35 02		44 26
...			
1935 Jun 26	93 11	45 38	
1927 Jun 28	95 06	45 34	
1919 Jul 1	98 00	45 31	
1937 Jul 2	99 37		45 31
1911 Jul 3	99 44	45 27	
1929 Jul 4	101 31		45 35
1903 Jul 5	102 03	45 23	
1921 Jul 7	104 25		45 39
1913 Jul 9	106 20		45 43
1905 Jul 12	109 13		45 48
...			
1938 Sep 8	166 24	44 32	
1932 Sep 10	168 49		47 40
1930 Sep 11	169 17	44 31	
1924 Sep 13	171 73		47 43
1922 Sep 14	172 11	44 32	
1916 Sep 15	173 37		47 46
1914 Sep 16	174 06	44 32	
1908 Sep 17	175 32		47 50
1906 Sep 19	176 59	44 33	
1900 Sep 19	177 26		47 52
...			

Date	Mean Sun	GEE	GWE
1935 Nov 16	234°08′		48°05′
1927 Nov 18	236 03		48 03
1919 Nov 20	237 57		48 00
1911 Nov 23	240 51		47 58
1903 Nov 25	242 46		47 55
1933 Nov 30	248 25	46°04′	
1925 Dec 2	250 19	46 09	
1917 Dec 5	253 13	46 15	
1909 Dec 7	255 08	46 19	
1901 Dec 10	258 01	46 25	
...			
1939 Jan 27	305 22		45 59
1931 Jan 29	307 17		45 54
1923 Feb 1	310 11		45 49
1915 Feb 3	312 05		45 44
1907 Feb 6	314 59		45 39
1937 Feb 9	318 40	47 57	
1929 Feb 11	320 34	47 58	
1921 Feb 13	322 29	48 00	
1913 Feb 16	325 23	48 01	
1905 Feb 18	327 17	48 02	

It is evident that Table 3 lacks information for large portions of the zodiac; to fill these gaps would require more than an additional 150 years of data. But even with these limitations, the table suggests that for any single position of the mean sun in the zodiac, there is one characteristic value of GEE and one characteristic value of GWE. In Proposition 9.3 below, Ptolemy will show that occasions on which GWE and GEE are equal correspond to positions of the mean sun that are equidistant from the planet's apogee or perigee. In the above table, note that 1919 Jul 1 and 1937 Jul 2 are two such occasions. We will consider them more fully in the Preliminaries to Book X.

The Planets and the Mean Sun

We noted earlier that the mean motion of Venus is equal to that of the mean sun; the same, moreover, is true for Mercury. Such equality is a necessary consequence of these planets' limited elongations from the mean sun. For if the mean motions of sun and planet were not equal, then over time the planet would move further and further away from the mean sun, eventually exceeding any limit. Ptolemy does not explain why Mercury and Venus should have any connection whatever to the mean sun, still less why that relation should be the special relation of equality. He simply notes it as a feature that characterizes them.

In contrast, Mars, Jupiter, and Saturn each appear at all elongations from the mean sun. Necessarily, their mean motions do not equal the mean sun's and are in fact far slower. Yet these planets too exhibit a definite relation to the mean sun; for each of them, the sum of its longitudinal and its anomalistic motions proves to be equal to the motion of the mean sun over the same period of time, as Ptolemy will note in Chapter 3. Again, Ptolemy simply states this relation as a characteristic of the three planets.

Thus there are two characteristic relations between the planets and the mean sun:

$$s = l + a$$

for Mars, Jupiter, and Saturn, where s is the motion of the mean sun, l the mean motion in longitude, and a the mean motion in anomaly; and

$$s = l$$

for Mercury and Venus.