Ptolemy's
ALMAGEST

Translated and Annotated by

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Duckworth

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Preface

A new English translation of the Almagest needs no apology. As one of the most influential scientific works in history, and a masterpiece of technical exposition in its own right, it deserves a much wider audience than can be found amongst those able to read it in the original. The existing English translation by R. Catesby Taliaferro,[1] besides being difficult to acquire, is such that silence is the kindest comment one can make. The French translation by N. Halma, virtually unobtainable, suffers from excessive literalness, particularly where the text is difficult. The other modern version, Karl Manitius' German translation, is on an entirely different level from these. It was done by a man who had studied the text and made a strenuous and on the whole successful effort to understand Ptolemy's meaning and methods. I have used it constantly for twenty years, and those to whom it is familiar will recognise how much I owe to it. Nevertheless, it is not free from mistakes, and, to my taste, errs in the direction of paraphrasing where it should be translating. Most important, one can no longer assume that those with a serious interest in history are able to read German with ease. I have been able to improve on Manitius' translation, in part because of work published since he made it, in part because I had independent access to much of the textual evidence, notably the mediaeval Arabic translations. I have drawn attention to a few passages where I have noticed that he is in error, but I have made no systematic comparison between my translation and his or any other version.

Every translator, and especially one dealing with an ancient language, is confronted with the dilemma of being faithful to the original and at the same time comprehensible to his readers. My intention was that this translation should serve both those who know no Greek, as a substitute for the text, and those who do, as an aid to reading it. This has inevitably led to compromises. On the whole, I have kept closely to the meaning and structure of the Greek, even, on occasion, where this entailed abandoning idiomatic English. But I have usually broken up Ptolemy's enormously long sentences (characteristic of Hellenistic scientific prose) into shorter units more suitable for English, and I have frequently substituted mathematical symbols (=, + etc.) and a symmetric presentation for the continuous rhetorical exposition of the ancient text. I have been liberal with explanatory additions, which are marked as such by enclosure within square brackets. Wherever the need to be intelligible forced me to a paraphrase, I give the literal translation in a footnote.

It would have made what is an already big book impossibly unwieldy if I had

[1] For full references here and elsewhere see the Bibliography.

provided a full technical and historical commentary on the Almagest. Fortunately two recent works, by Neugebauer (*HAMA*) and Pedersen, are excellent guides to the technical content, and the former is also of considerable help on the numerous historical problems which arise from it. I have therefore confined my own commentary to footnotes on points of detail (referring to the above works for expository treatments), and to an introduction giving the minimum of information necessary to understand and use the translation.

In the course of making the translation I recomputed all the numerical results in the text, and all the tables (the latter mostly by means of computer programs). The main purpose of this was to detect scribal errors (in which I have been moderately successful). But my calculations incidentally revealed a number of computing errors or distortions committed by Ptolemy himself. Where these are explicable as the result of rounding in the course of computation they are ignored, since to list some thousands of slightly more accurate results which I have found with modern mechanical aids would invite Ptolemy's own sardonic remark: 'Scrupulous accuracy about such a small amount is a sign of vain conceit rather than love of truth'. However, I have noted every computing error of a significant amount, and also those cases where the rounding errors are not random, but seem directed towards obtaining some 'neat' result. I hope that this will shed some light on the problem of Ptolemy's manipulation of his material (both computational and observational) in order to present an appearance of rigor in his theoretical treatment which he could never have found in his actual experience. The problem is an interesting one, which deserves an informed and critical discussion. Unfortunately, the recent book on this subject by R. R. Newton provides nothing of the kind, but rather tends to bring the whole topic into disrepute. The only detailed discussion which is useful is that by Britton [1].[2] This, however, is confined to certain classes of the observations. My own inferences from the computations tend to confirm Britton's conclusions about the nature and purpose of Ptolemy's manipulations of his data.

This book owes much to the help of numerous people and institutions, which I gratefully acknowledge here. The Bibliothèque Nationale, Paris, the Biblioteca Apostolica Vaticana and the Biblioteca de El Escorial provided me with microfilms of various Greek and Arabic manuscripts of the Almagest (detailed on pp. 3–4). I thank my colleague, David Pingree, Prof. Dr. Fuat Sezgin and Prof. Dr. Paul Kunitzsch for providing me with other microfilms and photocopies which I needed. Mr. Colin Haycraft not only gave me the necessary encouragement actually to embark on a project which I had been contemplating for a long time, but also bore patiently with the repeated delays until the book was ready for publication. When B. R. Goldstein, who was already engaged in preparing an English version of the Almagest, heard that I had decided to make this translation, he generously abandoned the project and turned over his materials to me. I owe to these and to him several ideas about format and notation. My pupil, Don Edwards, detected a number of slips and

[2] It is regrettable that this has never been formally published. It is available in Xerox copy from University Microfilms International, Ann Arbor, Michigan 48106.

typing errors in my preliminary version, and performed many useful services in comparing the translation with the Greek text. Michele Wilson drew Fig. F for me. Janet Sachs provided invaluable help in preparing the typescript for publication and eliminating numerous mistakes. Several of my footnotes on difficult problems have been influenced by my discussions with Noel Swerdlow. Rather than trying to disentangle his contribution at each place, I here record, with thanks, the stimulus he has given to my thinking. N. G. Wilson answered my questions on points of Greek palaeography and went out of his way to examine manuscripts at my request. My colleague, A. J. Sachs, gave me the benefit of his unrivalled expertise on several points of Babylonian astronomy and Mesopotamian history. To my colleague O. Neugebauer I owe more than I can express here. Let me say only that it was he who first introduced me to the Almagest more than twenty years ago, that his own investigations of it (only part of which have been published in his monumental *A History of Ancient Mathematical Astronomy*) have been invaluable to me as an aid and as a model, and that many will recognize his draughtsmanship in several of the supplementary diagrams. As an inadequate token I dedicate this book to him.

Providence, 1982 G.J.T.

Introduction

1. *Ptolemy*

For a detailed discussion of what little is known of the life of the author of the Almagest, and an account of his numerous other works, on astronomy, astrology, geography, optics and other mathematical subjects, I refer the reader to my article in the Dictionary of Scientific Biography (Toomer [5]). Here I mention only that his name was Claudius Ptolemaeus (Κλαύδιος Πτολεμαῖος), that he lived from approximately A.D. 100 to approximately A.D. 175, and that he worked in Alexandria, the principal city of Greco-Roman Egypt, which possessed, among other advantages, what was probably still the best library in the ancient world.

2. *The Almagest*

The Almagest is firmly dated to the reign of the Roman emperor Antoninus (A.D. 138–161). The latest observation used in it is from 141 February 2 (IX 7 p. 450), and Ptolemy takes the beginning of the reign of Antoninus as the epoch of his star catalogue (VII 4 p. 340). Although it is clear that Ptolemy had spent much time on it and that it is a work of his maturity (his own observations recorded in it range from A.D. 127 to 141), it has always been considered as his earliest extant work, because of the changes from it and references back to it in other works by him (for details see Toomer [5] p. 187). However, a recent discovery by Norman T. Hamilton (see IV n.51 p. 205) has shown that the 'Canobic Inscription' represents a stage in the development of Ptolemy's astronomical theory earlier than the Almagest. Since Ptolemy erected that dedication in the tenth year of Antoninus (A.D. 146/7), the Almagest can hardly have been published earlier than the year 150.

As is implied by its Greek name, μαθηματικὴ σύνταξις, 'mathematical systematic treatise', the Almagest is a complete exposition of mathematical astronomy as the Greeks understood the term. Whether there were any comparable works (i.e. *comprehensive* astronomical treatises) before it is not known. In any case, its success contributed to the loss of most of the work of Ptolemy's scientific predecessors, notably Hipparchus, by the end of antiquity, because, being obsolete, they ceased to be copied. Whereas Hipparchus' works are still used by Ptolemy's younger contemporaries, Galen and Vettius Valens,[1]

[1] E.g. Galen, *On Seven-month Children*, ed. Walzer 347, 350; Commentary on Hippocrates' *Airs Waters and Places* (see GAS VI 98). Vettius Valens, *Anthologiae* 354.

by the early fourth century (and probably much earlier),[2] when Pappus wrote his commentary on it, the Almagest had become the standard textbook on astronomy which it was to remain for more than a thousand years. Thus its importance for us lies not only in its value as a historical source for earlier theories and observations, but also, and perhaps chiefly, in its influence on all later astronomy in antiquity and the middle ages (in both Islamic and Christian areas) down to the sixteenth century. It was dominant to an extent and for a length of time which is unsurpassed by any scientific work except Euclid's *Elements*.

No attempt can be made here to sketch even an outline of the history of its influence.[3] I mention only some points to which I will make reference in the notes to the translation. The position of the Almagest as the standard textbook in astronomy for 'advanced students' in the schools at Alexandria (and no doubt at Athens and Antioch too) in late antiquity is amply demonstrated by the partially extant commentaries on it by Pappus (*c.* 320) and by Theon of Alexandria (*c.* 370). In the late eighth and ninth centuries, with the growth of interest in Greek science in the Islamic world, the Almagest was translated, first into Syriac, then, several times, into Arabic. In the middle of the twelfth century no less than five such versions were still available to the amateur ibn aṣ-Ṣalāḥ: a Syriac translation, two versions made under the Caliph al-Ma'mūn (an older one by al-Ḥasan ibn Quraysh, and one dated 827/8 by al-Ḥajjāj), a version by the famous translator Isḥāq ibn Ḥunayn (*c.* 879–90), and a revision of the latter by Thābit ibn Qurra (d. 901).[4] Two of these translations are still extant, those of al-Ḥajjāj and Isḥāq-Thābit. In them we find the title of Ptolemy's treatise given as 'al-mjsṭy' (consonantal skeleton only). This is undoubtedly derived (ultimately) from a Greek form μεγίστη (?sc. σύνταξις), meaning 'greatest [treatise]', but it is only later that it was incorrectly vocalised as al-majasṭī, whence are derived the mediaeval Latin 'almagesti', 'almagestum', the ancestors of the modern title 'Almagest'. The available evidence has been assembled and discussed by Kunitzsch, *Der Almagest* 115–25, where he makes a good case for supposing that the Arabic form was derived, not directly from the Greek, but from a middle Persian (Pahlavi) translation of the Almagest. There is independent evidence for the existence of the latter, but whether it was made as early as the reign of the Sassanid king Shahpuhr I (241–272), as later Persian accounts maintain, seems very dubious to me.

While Ptolemy's work in the original Greek continued to be copied and studied in the eastern (Byzantine) empire, all knowledge of it was lost to western

[2] The evidence for the practice of astronomy in the third century is pitifully small, but there exists a fragment of a text from about A.D. 213 which is closely related to the Almagest (see *HAMA* II 948–49), and there are several third-century papyri related to the Handy Tables (*ibid.* 974–75, 979–80). P. Ryl. 27 (written c. 260) quotes Ptolemy's solstice and equinox observations from Almagest III 1, and in the late third century Porphyry (*Comm. on Harmonica* 2, p. 24,15 ff.) quotes Almagest I 2 (H9, 11–16). The only evidence I have seen for knowledge of the Almagest in the second century, Galen, Commentary on Hippocrates' *Airs Waters and Places* III (ms. Cairo, Ṭal'at ṭibb 550, p. 73a), where Ptolemy is mentioned at the end of a list of authorities on astronomy, must be an interpolation in the Arabic tradition, since Ptolemy is there characterized as 'the king of Egypt'.

[3] I know of no satisfactory account of this. I gave a very brief sketch, Toomer[5] 202.

[4] For a full account of this see Kunitzsch, *Der Almagest*, especially 15–71. Kunitzsch has also published the work of ibn aṣ-Ṣalāḥ (see Bibliography).

Europe by the early middle ages. Although translations from the Greek text into Latin were made in mediaeval times,[5] the principal channel for the recovery of the Almagest in the west was the translation from the Arabic by Gerard of Cremona, made at Toledo and completed in 1175.[6] Manuscripts of the Greek text began to reach the west in the fifteenth century, but it was Gerard's text which underlay (often at several removes) books on astronomy as late as the Peurbach-Regiomontanus epitome of the Almagest (see Bibliography under Regiomontanus). It was also the version in which the Almagest was first printed (Venice, 1515). The sixteenth century saw the wide dissemination of the Greek text (printed at Basel by Hervagius, 1538), and also the obsolescence of Ptolemy's astronomical system, brought about not so much by the work of Copernicus (which in form and concepts is still dominated by the Almagest), as by that of Brahe and Kepler.

3. *The translation*

The basis of my translation is the Greek text established by Heiberg. I have, however, found it necessary to make several hundred corrections to that text. These are noted at the places in the translation where they occur,[7] and are also listed in Appendix B. In many cases (usually involving numerical computations), my correction consists of adopting the reading of the manuscript D, unjustly spurned by Heiberg as descended from an archetype due to an Alexandrian recension in late antiquity (Prolegomena, in Ptolemy, *Opera Minora* CXXVI–VII). Whatever the truth about that, and despite the fact that D itself is, as Heiberg says, 'most negligently written', I am convinced on grounds of internal consistency that it represents a sounder tradition than that of the mss. ABC, generally preferred by Heiberg. In many cases its obviously correct readings are shared by all or part of the Arabic tradition. Nevertheless, I have not deviated from Heiberg's text except where it seemed essential for sense or numerical consistency. In making corrections I have referred to photographs of the following manuscripts.

Greek (I use Heiberg's notation)

A Parisinus graecus 2389. Mainly uncial, ninth century

B Vaticanus graecus 1594. Minuscule, ninth century

D Vaticanus graecus 180. Several hands, but not, as Heiberg, *Almagest* I p. V, of the twelfth century, but rather of the tenth: see the Vatican Catalogue by Mercati and Franchi de' Cavalieri, I p. 206. N. G. Wilson has confirmed this dating for me by personal inspection. (Heiberg himself seems to have changed his opinion later: see Prolegomena LXXIX.)

Arabic (I have used the abbreviations 'Ar' to refer to the consensus of the

[5] See Haskins, *Studies* 103–112, 157–165.

[6] Kunitzsch, *Der Almagest* 83–112, gives a valuable account of the evidence for this, and of Gerard's method of work: evidently he used more than one of the Arabic translations.

[7] I have acknowledged there all cases known to me where my correction has been anticipated by others, notably Manitius.

Arabic tradition, and 'Is' to the consensus of the mss. containing the Isḥāq-Thābit version).

L Leiden, or. 680. Eleventh century according to Kunitzsch, *Der Almagest* 38. This is the only surviving manuscript of the version of al-Ḥajjāj.

T Tunis, Bibliothèque Nationale, 07116 (see Kunitzsch, *Der Almagest* 38–40). Completed October 1085. The Isḥāq-Thābit version, complete.

P Paris, B.N. ar. 2482. Completed December 1221. See Kunitzsch, *Der Almagest* 42–3. The Isḥaq-Thābit version, Books I–VI 13.

Q Paris, B.N. ar. 2483. Fifteenth century. See Kunitzsch, *Der Almagest* 43. The Isḥāq-Thābit version, Books I–VII.

E Escorial 914. See Kunitzsch, *Der Almagest* 43–4. The Isḥāq-Thābit version, Books V–IX.

F Escorial 915. Completed September 1276. See Kunitzsch, *Der Almagest* 44–5. The Isḥāq-Thābit version, allegedly containing Books VII–XIII, but in fact lacking large sections even of these, and bound in such disorder as to be almost useless.

Ger The Latin translation of Gerard of Cremona, for which I have used only the printed edition (Venice, Liechtenstein, 1515). For the complex dependence of this on the various Arabic versions see Kunitzsch, *Der Almagest* 97–104.

I did not undertake a complete collation of any of the above mss. For the Greek mss. that would have been largely useless, since Heiberg's reports are, as in all his editions, very accurate (to judge from my sporadic verifications; I remarked the rare exceptions in the notes to the translation). To collate the Arabic translation would have delayed this book for several years, with no commensurate gain. I have consulted the above mss. only in passages where I already considered Heiberg's text wrong or suspect. Therefore no conclusions should be drawn about the readings of the Arabic mss. where I do not explicitly report them.

There are a number of places where, if I were to establish a Greek text, it would differ from Heiberg's, but which I have not bothered to record in this book. Examples are:

mere orthography:

ηὑρίσκομεν	for εὑρίσκομεν (imperfect) I 327,15	
Κάλλιππος	for Κάλιππος	I 199,5
ἀμετάπειστον	for ἀμετάπιστον	I 6,18 (cf. Boll, *Studien* 74)
κρίκος	for κρῖκος	I 196,8

changes in form not affecting the sense: ἄν for ἐάν I 393,11
reversals of letters referring to figures: ZK for KZ I 243, 22
obvious misprints:

σελήνης	for σηλήνης	I 406,25
ἀνωμαλίας	for ἀμωμαλίας	I 462,19

(less obvious misprints, particularly those involving numbers, are recorded).

During the course of making the translation, I became convinced that the text contains quite a large number of interpolations, which must go back to antiquity, since they are in the whole manuscript tradition, both Greek and Arabic. I was first led to this conclusion by the discovery that there are places in the text, nonsensical as they stand, which can be made to yield perfect sense by the simple elimination of a clause or sentence, which must have been inserted as 'explanation' by someone who failed to understand Ptolemy's meaning. A notable example is V 1 (see p. 219 n.5). Cf also V 12, p. 245 with n.41. I later realised that there are whole classes of textual matter which must also be regarded as interpolations. One of these is the totals in the star catalogue (see pp. 16–17). The other is the chapter headings. Some of these (e.g. IX 2) are so inept as descriptions of the actual content of the chapter that it is impossible to attribute them to Ptolemy. In fact I do not believe that Ptolemy himself used any chapter divisions at all. It is obvious that he is responsible for the division into 13 books, both from the summaries that are found at the beginning of most books, and from explicit references such as 'in Book I' (ἐν τῷ πρώτῳ τῆς συντάξεως, II 1 p. 75) and 'in the preceding book' (ἐν τῷ πρὸ τούτων συντάγματι, VI 5 p. 283). But he never refers to a chapter division. Furthermore, there is some discrepancy in the manuscript tradition (especially between the branch represented by D and that represented by A) as to the points of division between chapters (e.g. at the beginning of Book III), and it is clear from Pappus' commentary that although a division into chapters already existed in his time, it was very different, at least in Book V, from the present division.[8] If the chapter division and headings are spurious, so too must be the table of contents preceding each book. Nevertheless, since this method of subdividing the text is useful for reference purposes, and appears in all editions, I have retained it, merely marking the character of the chapter headings by enclosing them in brackets thus: { }.

4. *What is in the Almagest, and what is not*

The order of treatment of topics in the Almagest (outlined in I 2) is completely logical. In Book I, after a brief treatment of the nature of the universe (in so far as it concerns the astronomer), Ptolemy develops the trigonometrical theory necessary for the work as a whole. In Book II he discusses those aspects of spherical astronomy which are related to the observer's position on earth (rising-times, length of daylight, etc.). Book III is devoted to the theory of the sun. This is a necessary preliminary for the treatment of the moon in Book IV, since the use of lunar eclipses there depends on one's ability to *calculate* the solar position. Book V treats the advanced lunar theory, which is a refinement of that in Book IV, and also lunar and solar parallax. Book VI is on eclipses, and thus requires a knowledge of both solar and lunar theory, and also of parallax. Books VII and VIII treat the fixed stars: since the moon is used as a 'marker' to determine the position of some crucial fixed stars, lunar theory must precede this, and since some planetary observations are made with respect to fixed stars,

[8] See the note in Rome[1] I p. 106, and cf. (for Theon) II p. 448 n. (1).

the establishment of a star catalogue (VII 5 and VIII 1) must precede the planetary theory. The last five books are devoted to the planets. Books IX–XI develop the theory of their longitudinal motion, Book XII treats retrogradations and greatest elongations (which depend only on longitude), while Book XIII deals with planetary latitude and those phenomena (the 'phases') which are partially dependent on it. Ptolemy occasionally anticipates later results for the sake of convenience (see IV 3 p. 179 and IX 3 p. 423, where the mean motion tables of moon and planets incorporate some later corrections), but in general the order of presentation, within books as well as in the treatise as a whole, is dictated by the logic of the didactic method.

There are, however, certain topics which Ptolemy does not discuss either because he takes it for granted that they are already known to his readers, or because it seemed superfluous to go into details (here I am referring especially to chronological matters). He says specifically (I 1 p. 37 with n.13) that the work is for 'those who have already made some progress in the field'. This means, in practice, that he assumes a knowledge of elementary geometry ('Euclid') and 'logistic' (thus he does not consider it necessary to explain how to extract a square root), and also of 'spherics'. The latter is illustrated by the extant works of Autolycus, Euclid (*Phaenomena*) and Theodosius (*Sphaerica*), which deal with the phenomena arising from the rotation of stars and sun about a central, spherical earth, e.g. their risings, settings, first and last visibilities, periods of invisibility etc., using elementary geometry, but arriving mainly at qualitative rather than quantitative results.[9] These results are mostly irrelevant to Ptolemy's work, but he does use much of the terminology and concepts of spherics without explanation.

5. *What the reader of the Almagest needs to know*

The modern reader, too, is likely to be familiar with elementary geometry. So I have not burdened the translation with references to Euclid except where the theorems assumed are not immediately obvious. However, in what follows I give a brief explanation of methods, concepts and facts not explained by Ptolemy which the reader of the Almagest needs to know, but which may be less familiar. On Ptolemy's mathematical methods in general one may profitably consult Pedersen 47–56.

(*a*) *The sexagesimal system*

This was taken over by the Greeks (one may guess by the Hellenistic astronomers) from the Babylonians as a convenient way of expressing fractions and (to a lesser extent) large numbers, and of performing calculations with them. It is the first place-value system in history. In the translation and notes I use the convenient modern 'comma and semi-colon' notation, in which

[9] For more detail see *HAMA* II 755–71.

6,13;10,0,58 represents $6\times 60+13+10\times 60^{-1}+0\times 60^{-2}+58\times 60^{-3}$. Ptolemy uses the system only for fractions, and represents whole numbers, even when combined with sexagesimal fractions, by the standard Greek (alphabetic) notation. The translation follows this mixed notation (thus the above number would be written 373;10,0,58 in the translation, and $\overline{\tau o\gamma}\ \bar{\iota}\ o\ \overline{\nu\eta}$ in Greek).

(*b*) *Fractions*

Except where it is necessary to be precise, Ptolemy prefers the traditional Greek fractional system to the sexagesimal. In this, although it is possible to express proper fractions as e.g. '4 5ths', preference is given to unit fractions, so that, e.g. '$\frac{3}{4}$' is expressed as the sum of $\frac{1}{2}$ and $\frac{1}{4}$ (written ∠′δ′, i.e. '$\frac{1}{2}\frac{1}{4}$'). There is a special sign for $\frac{2}{3}$. In the translation I have usually converted these sums of unit fractions to proper fractions without comment. However, I have always retained the fractional form where Ptolemy has it, since it gives a misleading appearance of precision to convert to sexagesimals (as Manitius often does, putting an exact number of minutes instead of a fraction of a degree). This is particularly true of the star catalogue.

(*c*) *Trigonometry*

The sole trigonometrical function used by Ptolemy is the chord. The derivation and structure of his chord table are fully explained in I 10. However, Ptolemy does not give explicit instructions for its use in trigonometrical calculations, although his method is obvious enough from the worked examples. In what follows I give a literal translation, with commentary, of a typical calculation involving trigonometry.

See Fig. A, and, for my conventions, compare the translation pp. 163–4. In the given situation arc ΘH is 30°, AD is 60^p, AH is $2;30^p$, and it is required to find the angle ADH (the 'equation'). In modern trigonometry we would use the cosine formula. Ptolemy has no equivalent, so he drops the perpendicular HK, thus transforming the problem into one of solving only right triangles, which is his standard procedure.[10]

'Then since arc ΘH is again 30 degrees, angle ΘAH would be 30 of those [units] of which 4 right angles are 360, and 60 of those [units] of which 2 right angles are 360. So the arc on HK is 60 of the units of which the circle [circumscribed] about the right-angled [triangle] HKA is 360, and the arc on AK is 120, the supplement making up the semi-circle. And so, of the chords subtended by them, HK will be 60 of the units of which hypotenuse AH is 120, and AK 103;55 of the same [units].'

[10] He knows the equivalent of the sine formula, namely that in the general triangle the sides are proportional to the chords of the doubles of the opposite angles, but uses it surprisingly infrequently. An example is IX 10 p. 462 (cf. n.96 there).

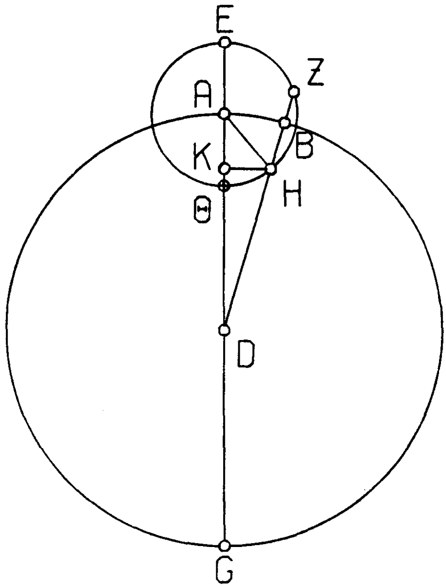

Fig. A

To solve a right-angled triangle (here HKA), Ptolemy imagines a circle circumscribed about it. Then the hypotenuse of the triangle is the diameter of the circle, and is taken (initially) as 120 parts (R = 60 being the standard on which Ptolemy's chord table is constructed). The two acute angles of the triangle being given, the other two sides can now be expressed in the same units: they are the chords of the arcs of the circumscribed circle, which are the doubles of the angles of the triangle (since they are equal to the angles at the centre). Instead of explicitly doubling these angles, Ptolemy always first expresses them in 'units of which 2 right angles are 360'. (Following the convention invented by B. R. Goldstein, I indicate these 'demi degrees' by the notation $^{\circ\circ}$, reserving $^{\circ}$ for the standard degree of which there are 90 in a right angle.) This enables him to switch smoothly from the triangle to the circle (and hence to the chord table, which gives him the actual numbers 60^p and $103;55^p$): an angle of size θ° is $2\theta^{\circ\circ}$, and hence the arc of the circumscribing circle which corresponds to that angle is $2\theta^{\circ}$.

'Therefore in those [units] of which line AH is 2;30, and the radius AD is 60, HK will be 1;15 and AK, likewise, 2;10, and KD, the remainder, 57;50.'

The sides of triangle AKH are converted to the norm representing their actual size (AH = $2;30^p$, hence they are multiplied by 2;30/120). This gives two sides of the next right triangle to be solved, DHK:HK and (by subtraction of AK from the given AD) KD.

'And since the squares on these added together make the square on DH, the latter will be, in length, approximately 57;51 of the units of which line KH was [found to be] 1;15.'

Since Ptolemy has no tangent function, he has to use 'Pythagoras' theorem' to find the hypotenuse of the right triangle in question. He uses the word μήκει, 'in length', to indicate that he is taking the square root (considered as the side of a square, hence a line length).

'And so of those [units] of which hypotenuse DH is 120, line HK will be 2;34 and the arc on it [HK, will be] 2;27 of those [units] of which the circle about DHK is 360. So that angle HDK is 2;27 of those [units] of which 2 right angles are 360, and about 1;14 of those of which 4 right angles are 360.'

The sides of triangle DHK are now converted to the standard in which the hypotenuse is 120^p, thus enabling Ptolemy to use the chord table to determine the size of the arc corresponding to the side opposite the angle to be determined, HDK. The latter, being at the circumference of the circumscribed circle, is half the arc. Ptolemy again expresses this relationship by saying that it is the same number of 'demi degrees' as the arc is 'single degrees', and then converting the 'demi degrees' to 'single degrees' by halving. Note that I frequently translate expressions like '30 degrees of the kind of which the great circle is 360' simply as '30°'.

(*d*) *Chronology and calendars*

Ptolemy's own chronological system is very simple. He uses the *Egyptian year* and the *era Nabonassar*. The Egyptian year is of unvarying length of 365 days, consisting of twelve 30-day months and 5 extra ('epagomenal') days at the end. Ptolemy uses the Greek transliterations of the Egyptian month names. For the reader's convenience, I usually add a Roman numeral indicating the number of the month. The order of the months is:

I	Thoth	VII	Phamenoth
II	Phaophi	VIII	Pharmouthi
III	Athyr	IX	Pachon
IV	Choiak	X	Payni
V	Tybi	XI	Epiphi
VI	Mechir	XII	Mesore.

The reason for choosing the era Nabonassar is given by Ptolemy at III 7 (p. 166: the earliest (Babylonian) observations available to him were from the reign of King Nabonassar. Ptolemy's epoch, Nabonassar 1, Thoth 1 corresponds to −746 February 26 in our reckoning.[11]

[11] Throughout this book I use the 'astronomical' system of dating according to the Christian era, since it is far simpler for calculating intervals than the 'B.C./A.D.' system. In this, year −1 corresponds to 2 B.C., year 0 to 1 B.C., year 1 to A.D. 1, etc.

Even when he refers to other calendars, Ptolemy usually gives the equivalent date in his own system, so there is no uncertainty. Sometimes, however, he gives, not the running date in the era Nabonassar, but only the regnal year of a king. It is clear that there already existed, in some form, a 'king-list' enabling one to relate the regnal year of a given king to a standard epoch.[12] Later, in his 'Handy Tables', Ptolemy published such a king-list (known as 'Canon Basileon'), and it survives, in a considerably augmented form, in Byzantine versions of Theon of Alexandria's revision of the Handy Tables. From these I have excerpted and 'reconstructed' the table on p. 11, which makes no historical pretensions, but is intended solely as an aid to readers of this book. The basis of the table is Usener's edition of the two versions in the manuscript Leidensis gr. 78, in *Monumenta Germaniae Historica, Auctores Antiquissimi* XIII (*Chronica Minora Saec. IV. V. VI. VII*, ed. Th. Mommsen), Vol. III, 447–53, supplemented by my own reading of the version in the ms. Vaticanus gr. 1291, 16^v–17^r. The names of the Babylonian and Assyrian kings are obviously very corrupt, and I have made no attempt to emend them, but have chosen those manuscript variants which seem closest to the forms now known from the cuneiform sources, which are listed in the second column (supplied to me by A. Sachs).

For the purposes of astronomical chronology, an integer number of years is assigned to each reign. As far as can be checked from independent sources, 'Year 1' of each reign was assumed to begin on the Thoth 1 *preceding* the historical date on which the king began to reign.[15] Thus, to use the table to go from a given regnal year to the era Nabonassar, one simply adds the number of the regnal year to the total listed (in the fourth column) for the previous king.[16] E.g. to find the second year of Mardokempad in the era Nabonassar (cf. IV 8 p. 204), we add 2 to the total of 26 given for his predecessor, Ilulai, and get the twenty-eighth year in the era Nabonassar.

Although I supply in the translation the modern equivalent of all dates in the Almagest, I have added, for the use of those readers who wish to check them, a fifth column listing the Julian equivalent of the first day of each king's reign. If one bears in mind that every Julian year divisible by 4 is a leap-year, while the Egyptian year is constant, this is a sufficient basis for the calculation. However, I recommend as an easier alternative the use of Schram's *Kalendariographische Tafeln*: from pp. 182–9 of that one can find the Julian day number of any date in

[12] Papyrus fragments of such king-lists are found in P. Oxy. 1.35 and Sattler, *Studien* 39–50. These are, however, later than Ptolemy. P. Oxy. 19.2222, a list of the Ptolemies of Egypt, is earlier than the Almagest, but is very different in format from Ptolemy's king-list.

[13] It is not known why these two kings are combined. In cuneiform sources (e g. the king-list translated in Pritchard, *Ancient Near Eastern Texts* 272 (iv), they appear consecutively, Ukīn-zēr being assigned 3 years and Pūlu 2.

[14] This must be a corruption in the Greek tradition of Arses ('Αρσης), the usual form of this king's name (also known as 'Οαρσης).

[15] This was recognised long ago. See Usener, *MGH* XIII.3 p. 441, with references to older literature in his n.5.

[16] In the Handy Tables Ptolemy adopted the 'era Philip' (which already occurs in the Almagest as 'death of Alexander'); hence in the mss. the totals for era Nabonassar go only as far as Alexander the Macedonian (no. 31), and a new totalling system begins with Philip (no. 32). I have converted all these later totals to the era Nabonassar by the addition of 424 to each. Cf. Schram p. 173.

	Ruler	Correct form	Years of reign	Total years to end of reign	Julian date of beginning of reign
	Kings [of Assyria and Babylonia]				
1	Nabonassar	Nabû-nāṣir	14	14	-746 Feb. 26
2	Nadi	Nādin	2	16	-732 Feb. 23
3	Chinzer and Por[13]	Ukīn-zēr; Pūlu	5	21	-730 Feb. 22
4	Ilulai	Elūlai	5	26	-725 Feb. 21
5	Mardokempad	Marduk-apla-iddin	12	38	-720 Feb. 20
6	Arkean	Šarru-ukīn	5	43	-708 Feb. 17
7	First interregnum		2	45	-703 Feb. 15
8	Belib	Bēl-ibni	3	48	-701 Feb. 15
9	Aparanad	Aššur-nādin-šumi	6	54	-698 Feb. 14
10	Regebel	Nergal-ušēzib	1	55	-692 Feb. 13
11	Mesesemordak	Mušēzib-Marduk	4	59	-691 Feb. 12
12	Second interregnum		8	67	-687 Feb. 11
13	Asaridin	Aššur-aḫa-iddina	13	80	-679 Feb. 9
14	Saosdouchin	Šamaš-šuma-ukīn	20	100	-666 Feb. 6
15	Kiniladan	Kandalānu	22	122	-646 Feb. 1
16	Nabopolassar	Nabû-apla-uṣur	21	143	-624 Jan. 27
17	Nabokolassar	Nabû-kudurra-uṣur	43	186	-603 Jan. 21
18	Illoaroudam	Amīl-Marduk	2	188	-560 Jan. 11
19	Nerigasolassar	Nergal-šarra-uṣur	4	192	-558 Jan. 10
20	Nabonadi	Nabû-na'id	17	209	-554 Jan. 9
	Kings of the Persians				
21	Cyrus	Kūruš	9	218	-537 Jan. 5
22	Kambyses	Kambuživa	8	226	-528 Jan. 3
23	Darius I	Dārayavaʰu	36	262	-520 Jan. 1
24	Xerxes	χšayāršā	21	283	-485 Dec. 23
25	Artaxerxes I	Artaχšaθrā	41	324	-464 Dec. 17
26	Darius II	Dārayavaʰu	19	343	-423 Dec. 7
27	Artaxerxes II	Artaχšaθra	46	389	-404 Dec. 2
28	Ochus	Vahauka	21	410	-358 Nov. 21
29	Arogos[14]	?Hawarša	2	412	-337 Nov. 16
30	Darius III	Dārayavaʰu	4	416	-335 Nov. 15
31	Alexander the Macedonian	'Αλέξανδρος	8	424	-331 Nov. 14
	Kings of the Macedonians				
32	Philip who succeeded Alexander the founder	Φίλιππος	7	431	-323 Nov. 12
33	Alexander II	'Αλέξανδρος ἕτερος	12	443	-316 Nov. 10
34	Ptolemy son of Lagos	Πτολεμαῖος Λάγου	20	463	-304 Nov. 7
35	Ptolemy Philadelphos	Φιλάδελφος	38	501	-284 Nov. 2
36	Ptolemy Euergetes	Εὐεργέτης	25	526	-246 Oct. 24
37	Ptolemy Philopator	Φιλοπάτωρ	17	543	-221 Oct. 18
38	Ptolemy Epiphanes	'Επιφανής	24	567	-204 Oct. 13
39	Ptolemy Philometor	Φιλομήτωρ	35	602	-180 Oct. 7
40	Ptolemy Euergetes II	Εὐεργέτης β'	29	631	-145 Sept. 29
41	Ptolemy Soter	Σωτήρ	36	667	-116 Sept. 21
42	Ptolemy Neos Dionysus	Διόνυσος νέος	29	696	-80 Sept. 12
43	Cleopatra	Κλεοπάτρα	22	718	-51 Sept. 5
	Kings of the Romans				
44	Augustus	Augustus	43	761	-29 Aug. 31
45	Tiberius	Tiberius	22	783	14 Aug. 20
46	Gaius	Gaius	4	787	36 Aug. 14
47	Claudius	Claudius	14	801	40 Aug. 13
48	Nero	Nero	14	815	54 Aug. 10
49	Vespasian	Vespasianus	10	825	68 Aug. 6
50	Titus	Titus	3	828	78 Aug. 4
51	Domitian	Domitianus	15	843	81 Aug. 3
52	Nerva	Nerva	1	844	96 July 30
53	Trajan	Traianus	19	863	97 July 30
54	Hadrian	Hadrianus	21	884	116 July 25
55	Antoninus	Aelius Antoninus	23	907	137 July 20

the era Nabonassar in a few seconds, and hence (from his other tables) the equivalent date in any standard calendar.

The only other aspect of Ptolemy's own chronology requiring remark is the 'double dates'. He frequently characterises the day of an observation by expressions like Παχὼν ιζ′ εἰς τὴν ιη′, translated 'Pachon 17/18', but literally 'Pachon, the seventeenth towards the eighteenth'. Modern commentators have made unnecessarily heavy weather of this. Ptolemy himself uses a noon epoch, but this is an artificial starting-point (the reason for which he explains at III 9 pp. 170–1), and has nothing to do with *numbering* the day. In antiquity the 'civil epoch' of the day was either dawn (as in Egypt) or sunset (as in Babylon). In either system, an event which took place in the daylight would be on the same 'day', but one which took place in the night would be on 'day n' for those using dawn epoch and 'day n+1' for those using sunset epoch. Hence ambiguity was possible. Ptolemy uses double dates (which are found *only* for night-time observations) to avoid this ambiguity. The form he uses implies the Egyptian, i.e. dawn epoch (cf. the long form III 1 p. 138,τῇ ια′ τοῦ Μεσορὴ μετὰ β̄ ὥρας ἐγγὺς τοῦ εἰς τὴν ιβ′ μεσονυκτίου (literally 'on the eleventh of Mesore, approximately two hours after the midnight towards the twelfth'), but it would be clear even to someone using sunset epoch (who would date the above event to 'Mesore 12') what day he means.

In using the observations of his predecessors Ptolemy often has occasion to refer to other systems of chronology and calendars. Although in such cases one can always readily derive the equivalent date in Ptolemy's own system (he almost always gives it explicitly), I shall describe them briefly here.

The most frequently mentioned is the *Kallippic Cycles*. To explain this, we must go back to Meton, who in −431 devised a 19-year 'cycle', i.e. a fixed scheme of intercalation of months containing 6940 days (thus the average length of a year was $365\frac{1}{4} + \frac{1}{76}$ days).[17] Since he was an Athenian, he used the month names of the Athenian civil calendar for the months of his artificial 'calendar'. A hundred years later an associate of Aristotle, Kallippos, produced a revision of this, based on the more accurate year-length of $365\frac{1}{4}$ days. In order to achieve this, he eliminated one day from 4 Metonic cycles, thus producing the 'Kallippic cycle' of 76 years and 27759 days. What was later known as the 'First Kallippic Cycle' began at the summer solstice (probably June 28th) of the year −329. In the Almagest we find references also to the Second and Third Kallippic Cycles, which began in −253 and −177 respectively. To judge from the Almagest, this chronological system was the one most used by earlier Hellenistic astronomers.[18] In VII 3 four observations by Timocharis (Alexandria, third century B.C.) are given according to the year of the First Kallippic Cycle and 'Athenian' month and day. On the basis of these, several attempts have been made to reconstruct the whole 'Kallippic calendar', with discrepant results. Since the above constitute the whole evidential basis, apart from the

[17] For a detailed discussion see Toomer[7]. I give there the arguments for supposing that Meton's purpose was not to reform the Athenian calendar, but to establish an 'astronomical chronology'.

[18] The dates of the three eclipses in IV 11 (p. 211, cf. n.63 there) which, though observed in Babylon, are given according to Athenian archon and Athenian month, are presumably in the Metonic calendar.

passage in Geminus, *Eisagoge* VIII, which I regard as fiction, and two dubious equivalences in the Milesian parapegma, any reconstruction is academic.[19] Here I note only that Kallippos evidently retained the peculiar Athenian method of counting the days of the month by decads, and in the last decad counting backwards, so that VII 3 p. 336 τῇ ς′ φθίνοντος, literally 'on the sixth [day] of the waning [moon]', means 'the sixth day from the end of the last decad', i.e. the twenty-fifth.[20]

Hipparchus too used the Kallippic cycles for astronomical dating, but combined them, not with Kallippos' 'Athenian' calendar, but with the Egyptian calendar (i.e. he used the cycles simply as a year count), at least as far as we can tell from the Almagest. This seems to have led to ambiguities, since the 'Kallippic' year began at or near the summer solstice, while the Egyptian year is a 'wandering year', which in Hipparchus' time began about the end of September. Thus there arose the possibility of a discrepancy of 1 in the year count, for certain stretches of the year (whether it is +1 or −1 depends on Hipparchus' choice). Such a discrepancy is firmly attested in Almagest IV 11 (see p. 214 n. 72), and cannot plausibly be removed by emendation, though this has been done (by Ideler and others) in the interest of consistency. In fact it is impossible to make all of Hipparchus' 'Kallippic cycle' dates in the Almagest consistent with one another (see p. 224 no. 13), and we must allow for the possibility that Hipparchus used different systems in different works.

Three planetary observations in the Almagest are dated κατὰ Χαλδαίους, 'according to the Chaldaeans', with a year number and a Macedonian month name and day number. The year numbers show that the era used is that known in modern times as the *Seleucid Era* (dating from the year which Seleucus I counted as the first of his reign, −311/10), which was common throughout the Seleucid empire. Since the observations are undoubtedly Babylonian, the particular epoch used in them is, as one would expect, that known from the surviving Babylonian astronomical texts, 1 Nisan (April) −310 (Greeks under the Seleucid empire commonly used an epoch of autumn −311). The use of Macedonian month names has rightly been taken to show that the Babylonian lunar months were simply called by the names of the Macedonian months by the Greeks under the Seleucid empire: if one computes the date of the first day of the 'Macedonian' month from the equivalent date in the era Nabonassar given by Ptolemy, it coincides (with an error of no more than one day) with the computed day of first visibility of the lunar crescent at Babylon.[21] There is other evidence for the assimilation of the month names,[22] but this is the strongest.

Unattested outside the Almagest is the *Calendar of Dionysius*. This had a

[19] Those who care to may consult Ginzel II 409–19 and Samuel, *Greek and Roman Chronology*, 42–9 for details and literature.

[20] For this system see Samuel, *Greek and Roman Chronology* 59–60. I do not know why it is not used for the other three 'Kallippic' dates in which the days are simply numbered consecutively.

[21] These are conveniently listed in Parker-Dubberstein.

[22] For details see Samuel, *Greek and Roman Chronology* 140–2. However, Samuel is wrong in saying that the Almagest evidence proves that the assimilation was made as early as the date of the earliest observation (Nov. −244). In the cuneiform record from which this was derived the Babylonian names must have been used. It was only when this was translated into Greek (which may have been as much as a century later) that the Macedonian names were substituted.

running year count and months named after the signs of the zodiac (corresponding, at least approximately, to the period of the year when the sun was in the sign in question). The months Tauron (♉), Didymon (♊), Leonton (♌), Parthenon (♍), Skorpion (♏), Aigon (♑) and Hydron (♒) are attested. From analysis of the Almagest evidence Böckh, *Sonnenkreise* 286–340, showed that the epoch of the calendar was the summer solstice of -284. Since Thoth 1 (Nov. 2) of -284 is the beginning of the first regnal year of Ptolemy Philadelphos, it is plausibly concluded that Dionysius observed in Egypt. Böckh's further conclusions, that the calendar was similar to the Egyptian one in having 12 months of 30 days, but was modified by introducing a sixth epagomenal day every four years, cannot be regarded as certain, especially since this requires 'emending' some of the Almagest dates. Here, as for the Kallippic calendar, 'reconstruction' seems pointless when the evidence is so scanty and the likelihood of verification utterly remote.[23]

One observation is dated in the *Bithynian calendar* of the imperial period. Like a number of other contemporary calendars in Asia Minor, this was simply the Julian calendar, with different month-names, and with the first day of the year Augustus' birthday, Sept. 23. For details and literature see Samuel, *Greek and Roman Chronology* 174–5.

(*e*) *Ptolemy's star catalogue*

The list of the coordinates and magnitudes of the principal fixed stars visible to Ptolemy poses special problems to the translator. In particular, there are numerous manuscript variants in the coordinates, and while one must put some number in the translation, it is often difficult to be certain about one's choice. The solution I have adopted is (in the star catalogue only) to append an asterisk to any element (longitude, latitude, magnitude, description or identification) where there is reason to suppose that it may be incorrect (i.e. not what Ptolemy wrote or intended),[24] either because there is a plausible ms. variant, or because of some gross inconsistency with the astronomical facts. In such cases I give all significant variants known to me in a footnote. I have made no effort to record all variants, since most are obviously wrong. The reader who wishes to go further must still consult Peters-Knobel, on which I have drawn heavily, and which is still the best treatment of the catalogue as a whole, though badly in need of updating and revision in certain respects.[25]

Ptolemy lists the stars under 48 constellations, and gives for each star (1) a description of its location on the 'figure' and (sometimes) of its brightness and colour; (2) its longitude; (3) its latitude and direction (north or south of the ecliptic); and (4) its magnitude. I have followed my predecessors (notably Manitius) in adding to these: (a) an initial column giving a running number to

[23] The interested reader may consult *HAMA* III 1067 n.2 and Samuel, *Greek and Roman Chronology* 50, n.6 for further literature.

[24] The lack of an asterisk does not imply that I regard the reading adopted as Ptolemy's beyond any question, but only that I have no good reason to doubt it.

[25] See the strictures of Kunitzsch, *Der Almagest* 46.

the star within its constellation (stars listed at the end of some constellations by Ptolemy as 'outside the constellation', i.e. not part of the imaginary figure, are numbered continuously with those preceding them); (b) a final column giving the modern identification of the star. For those stars which have them, this is the Bayer letter or Flamsteed number. Certain fainter stars have neither; for these I give the number in the Yale Bright Star Catalogue (abbreviated as 'BSC'). From that publication those interested can find the corresponding number in the Durchmusterung and the Henry Draper and Boss General Catalogues. I have abandoned all references to the antiquated Piazzi catalogue (still used by Peters-Knobel).

I have used Roman numerals to number the constellations, and refer to individual stars (throughout the translation) by the combination of Roman and Arabic numerals (thus 'catalogue XXXIX 2' refers to the second star in the thirty-ninth constellation (Canis Minor), namely Procyon).

The star descriptions pose numerous individual problems, only a few of which are touched on in the footnotes. Ideally one should provide a reconstruction of the outline of each constellation as it appears on Ptolemy's star-globe. Unfortunately no one has done the necessary work of assembling and comparing all the literary and iconographic evidence from antiquity and from the derivative Arabic tradition (notably aṣ-Ṣūfī). This would be an interesting and valuable enterprise. Meanwhile, for the reader who needs some visual illustration, I can recommend only the old work of Bayer, *Uranometria*, with the warning that in many cases his positioning of the stars on the figures, and the outlines of the figures themselves, are certainly different from Ptolemy's.[26] On the matter of the orientation of the figures, I have satisfied myself that Ptolemy describes them as if they were drawn on the *inside* of a globe, as seen by an observer at the centre of that globe, and facing towards him. This is in agreement with what Hipparchus says (*Comm. in Arat.* I 4 5): 'for all the stars are described in constellations (ἠστέρισται) from our point of view, and as if they were facing us, except for such of them as are drawn in profile' (κατάγραφον, as interpreted by Manitius, whom I follow dubiously). It is in this sense that we must interpret 'left hand', 'right leg', etc. This has to be said, since on the actual star globes the constellations were necessarily drawn on the *outside*. Hence the orientation of the figures was (at least in some cases) reversed, which could lead to confusion.[27] I have rendered the prepositions used by Ptolemy in indicating the positions of stars with respect to parts of the figures consistently, as follows:

in = ἐν
on = ἐπί
over = ὑπέρ

[26] The work of Thiele, *Antike Himmelsbilder*, is very little help, although I have referred to it to illustrate some particulars.

[27] Cf. the scholion on Aratus, Maass, *Comm. in Arat.* p. 384 no. 251: 'the signs look inward with respect to the heavens . . . but they have their backs to the globe, so that their faces may be seen. Hence, if he says "right hand" or "left hand" and we find the opposite on the globe, we should not be confounded.'

above = ἐπάνω
under = ὑπό
below = ὑποκάτω
just over = κατά + genitive
advance, in advance = προηγούμενος
rear, to the rear = ἑπόμενος

On the meaning of the last two terms see below p. 20. Note that 'rear' is never used in a sense other than directional. To indicate the back parts of an animal figure I use 'hind'.

Both longitudes and latitudes are given, not in degrees and minutes, but in degrees and fractions of a degree. I have retained this in the translation (see p. 7). With very few exceptions, the longitudes are not given more accurately than to $\frac{1}{6}°$. (This has been taken to imply that the ecliptic ring of Ptolemy's instrument was graduated only every 10′). However, one frequently finds the fractions $\frac{1}{4}°$ and $\frac{3}{4}°$ for the latitudes.

The latitudes in Ptolemy's list are preceded by the direction (βο = βόρειος, 'northern'; νο = νότιος, 'southern'). I have rendered these by + and − respectively.

The magnitudes range (according to a system which certainly precedes Ptolemy, but is only conjecturally attributed to Hipparchus) from 1 to 6. Ptolemy indicates intermediate magnitudes by adding (after the number) μείζων, 'greater' or ἐλάσσων, 'less' (abbreviated in the mss.). I have rendered these by > and < (before the number) respectively. One occasionally finds for the magnitude, instead of a number, the remark ἀμαυρός (rendered 'f.' for 'faint') or νεφελ. (for νεφελοειδής), 'nebulous', abbreviated as 'neb.'

For the identifications, wherever Peters-Knobel and Manitius are in agreement, I have usually been content to adopt their opinion. Where they differ (and even when they agree, in some special cases),[28] I have checked the possibilities as carefully as I could, using the large-scale *Atlas of the Heavens* by Bečvář, and transforming Ptolemy's coordinates to right ascension and declination at the modern epoch, where necessary. However, I have made no attempt to redo the work of Peters and Knobel, namely to compute the longitude and latitude of the relevant stars for Ptolemy's time from modern data (in particular using the most up-to-date values for the proper motions). This might be worth while, though I doubt whether the degree of improvement over Peters-Knobel would justify the large amount of computation. In any case, it is unlikely that it would eliminate the doubts that remain about the identification of many of the fainter stars.

At the end of each constellation in the mss. are listed the total number of stars in the constellation, and the sub-totals of each magnitude. These in turn are added up at various intermediate points (the northern segment, the zodiac, and the southern segment), and the grand totals are given at the end. I am

[28] Notably, where Ptolemy describes a star as a 'nebulous mass' (νεφελοειδὴς συστροφή), I have preferred to give the globular cluster (abbreviated 'CGlo') or galactic cluster (abbreviated 'CGal') rather than some particular star inside it.

convinced that this was not done by Ptolemy (who makes no mention of it in his description of the catalogue, VII 4 pp. 339–40). Another indication of the spuriousness of these passages is that no separate count is made in the totals of the stars which are greater (>) or less (<) than a certain magnitude: all are lumped in with the stars of that magnitude. I have translated the passages in question, but enclosed them in brackets thus: { }.

(*f*) *Explanations of special terms*

(i) *Geometrical*

by subtraction (λοιπός -ή -όν): literally 'the remaining [part]', 'remainder' (I have on occasion so rendered it).

by addition (ὅλος -η -ον): literally 'the total'.

Crd x: chord of the angle x° (R = 60^p). Greek has no word with the specific meaning 'chord', but uses the generic εὐθεῖα, 'straight line'. 'Crd x' renders ἡ τὰς x μοίρας ὑποτείνουσα εὐθεῖα, 'the straight line subtending x degrees'.

In connection with the Menelaus Theorem (see p. 18), an expression of the type 'Crd arc 2AB' represents ἡ ὑπὸ τὴν διπλῆν τῆς AB περιφερείας, literally 'the [line] subtended by the double of arc AB'.

supplement, supplementary arc (ἡ λείπουσα [λοιπὴ] εἰς τὸ ἡμικύκλιον περιφέρεια): literally 'the arc which is the remainder to the semi-circle'.

complement (λοιπὴ εἰς τὸ τεταρτημόριον): literally, 'the remainder to the quadrant'.

|| literally, 'is similar to'. Used of arcs of different-sized circles. Arc AB|| arc GD if each arc is the same fraction of its circle.

||| (ἰσογώνιόν ἐστι): literally, 'has [all] its angles equal to', i.e. is similar to (used only of triangles).

≡ (ἰσόπλευρόν ἐστι): literally 'has its sides equal to', i.e. is congruent to. Used only of spherical triangles. Sometimes ἰσογώνιον καὶ ἰσόπλευρόν ἐστι, 'has its angles and sides equal to'.

Q.E.D. (ὅπερ ἔδει δεῖξαι): literally 'which is what it was required to prove'.

componendo (συνθέντι). Expresses the operation of addition of ratios: if a : b = c : d, then (a + b):b = (c + d):d.

dividendo (διελόντι, κατὰ διαίρεσιν) (1) Usually expresses the operation of subtraction of ratios: if a : b = c : d, then (a − b) : b = (c − d) : d.

(2) Once, at XII 1 (see p. 558 n.4) διελόντι expresses *division* of members of ratios. If a : b = c : d, then $\frac{a}{n}$: b = $\frac{c}{n}$: d.

Menelaus Configuration and *Menelaus Theorem* (used only in the footnotes and explanatory additions). Cf. *HAMA* 26–9. Fig. B represents a Menelaus Configuration. m,n,r and s are four great circle arcs on the surface of the sphere, intersecting each other as shown, and divided by the intersections into the parts m_1, m_2 etc. (thus $m = m_1 + m_2$ etc.) In I 10 Ptolemy proves the theorems

$$\text{I} \qquad \frac{\text{Crd } 2m}{\text{Crd } 2m_1} = \frac{\text{Crd } 2r}{\text{Crd } 2r_1} \times \frac{\text{Crd } 2s_2}{\text{Crd } 2s}$$

$$\text{II} \qquad \frac{\text{Crd } 2r_2}{\text{Crd } 2r_1} = \frac{\text{Crd } 2m_2}{\text{Crd } 2m_1} \times \frac{\text{Crd } 2n}{\text{Crd } 2n_2}.$$

Since it is known that these were discovered by Menelaus, Neugebauer has named them 'Menelaus Theorem I' and 'Menelaus Theorem II' respectively, and I follow him, abbreviating to 'M.T.I.' and 'M.T.II'.

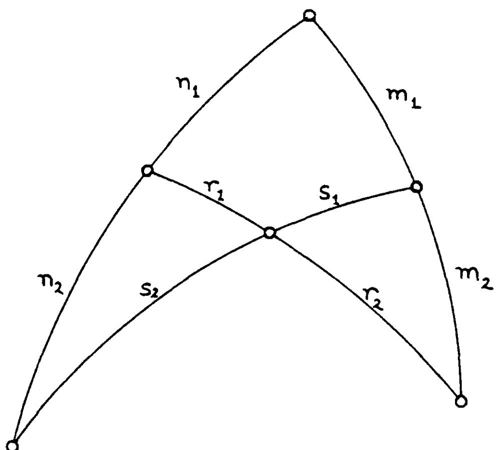

Fig. B

(ii) *Spherical astronomy*

(*at*) *sphaera recta* (ἐπ' ὀρθῆς τῆς σφαίρας) and (*at*) *sphaera obliqua* (ἐπ' ἐγκεκλιμένης τῆς σφαίρας). These mediaeval Latin terms are the literal translations of the Greek, meaning 'on the upright sphere' and 'on the inclined sphere' respectively. Probably taken from the use of celestial globes, they refer to the phenomena which occur when the celestial equator is perpendicular to the local horizon (*sphaera recta*) or inclined to it at an acute angle (*sphaera obliqua*). In particular, we use *rising-time at sphaera recta* or *right ascension*, and *rising-time at sphaera obliqua* or *oblique ascension* to designate the arc of the equator which crosses the horizon together with a given arc of the ecliptic (e.g. one zodiacal sign) at *sphaera recta* (i.e. at the terrestrial equator), and at *sphaera obliqua* (i.e. any other terrestrial latitude) respectively.

equator represents ἰσημερινὸς (κύκλος), literally 'circle of equal day', so called for the reason Ptolemy gives in I 8 (pp. 45–6).

meridian represents μεσημβρινὸς (κύκλος), literally 'midday circle' (defined and explained at I 8 p. 47). Meridian passage of a heavenly body is called *culmination*. The Greek terms for culminate and culmination, μεσουρανεῖν, μεσουράνησις, mean literally 'being in the middle of the heaven'. *upper* and *lower* culmination are expressed by ὑπὲρ γῆν and ὑπὸ γῆν, meaning 'above the earth' and 'below the earth' respectively, and sometimes so translated.

An *altitude circle* is any circle drawn through the zenith perpendicular to the horizon. Ptolemy has no special term for this in the Almagest, merely saying 'the (great) circle drawn through the zenith (through the poles of the horizon)', e.g. II 12, HI 166, 20–1.

colure. This term is used by Ptolemy only once, at II 6 p. 83. I translate part of Manitius' note on that passage: Two of the circles of declination through the poles of the equator are named 'colure' (κόλουρος): the solsticial colure, which goes through the solstices and hence carries the poles of the ecliptic, and the equinoctial colure. These two colures divide the sphere into four equal parts and divide both ecliptic and equator into four quadrants, so that one quadrant corresponds to each season of the year. Ptolemy counts the solsticial colure as boundary of the daily revolution [I 8 pp. 46–7, where however the term 'colure' is not used], but never explicitly mentions the equinoctial colure. Both colures were already defined by Eudoxus (Hipparchus, *Comm. in Arat.* 117 ff.) The term is explained by Achilles, *Isagoge* 27 (Maass, *Comm. in Arat.* 60) as follows: 'They are called colures because they appear to have their tails cut off as it were (κεκολοῦσθαι ὥσπερ τὰς οὐράς), since we cannot see the parts of them beginning at the antarctic, always invisible parallel'.

It is unfortunate that we have to use the same word *latitude* to refer both to the celestial coordinate (vertical to the ecliptic) and to the unrelated terrestrial coordinate. Ptolemy uses, for the former πλάτος, and for the latter κλίμα, literally 'inclination'. When necessary I gloss this e.g. as '[terrestrial] latitude'. κλίμα, however, does not refer to the coordinate as such (for which Ptolemy uses ἔγκλιμα, HI 68,9, ἔγκλισις, HI 101,23 or, once, πλάτος, HI 188,4), but to a specific 'band' of the earth where the same phenomena (e.g. length of longest daylight) are found. Hence in early Hellenistic times arose the notion of the division of the known world (the οἰκουμένη) into 7 standard *climata* (see *HAMA* 334 ff., II 727 ff. and Honigmann, *Die sieben Klimata*). This is reflected in several places in the Almagest, e.g. in Table II 13. I refer to these seven standard parallels by Roman numerals, e.g. Clima IV = the parallel through Rhodes, longest day 14½ hours.

(iii) *Referring to the heavenly bodies*

As Ptolemy explains in I 8, in his system the whole heavens are conceived as rotating from east to west, making one revolution daily. The direction defined by this motion, and the direction counter to it, are called εἰς τὰ προηγούμενα ('towards the leading [parts]') and εἰς τὰ ἑπόμενα ('towards the following [parts]') respectively. The corresponding adjectives προηγούμενος and ἑπόμενος are also found, particularly in the star catalogue, and Ptolemy frequently uses the phrases εἰς τὰ προηγούμενα (ἑπόμενα) τῶν ζῳδίων, 'towards the leading (following) [parts] of the zodiacal signs', to indicate the direction of motion in the ecliptic. A modern reader may find this confusing: since the normal motion of bodies in the ecliptic is from west to east, what we regard as forward motion, e.g. of a planet, is described as 'towards the following [parts]' ('towards the rear' in my translation). No version of these terms in a modern language is satisfactory. One cannot use 'west' and 'east' because these must be reserved for Ptolemy's δυσμαί and ἀνατολαί, which are confined to situations where a terrestrial observer is implied. It is a distortion to translate (with Manitius) 'in the reverse order of the signs' and 'in the order of the signs', since this implies that the terms define *ecliptic* coordinates, whereas they are in the *equatorial* system, and while it is usually true that a celestial object which προηγεῖται ('leads') another will have a lesser ecliptic longitude, if their latitudes differ greatly the reverse may be true, especially at very high ecliptic latitudes. Precisely this situation occurs in the star catalogue, despite Ptolemy's own statement at VII 4 p. 340 that the terms in the catalogue define ecliptic coordinates (see n.93 there). Although I am aware that my choice too has its drawbacks, I have settled on *in advance* for εἰς τὰ προηγούμενα, and *towards the rear* for εἰς τὰ ἑπόμενα. These always imply 'with respect to the daily motion from east to west', with the paradoxical consequence, as remarked above, that in the ecliptic a body which is 'in advance' of another has a lesser longitude. However, I have committed an inconsistency in translating the derived noun προήγησις as *retrogradation*. This is used only for the portion of the courses of the five planets in which they reverse their normal direction of motion, and it would be too confusing to render this by 'motion in advance'.

ecliptic. Ptolemy never refers to this circle by the term ἐκλειπτικός (which he confines strictly to the meaning 'having to do with eclipses'). His normal term is ὁ διὰ μέσων τῶν ζῳδίων (κύκλος), 'the (circle) through the middle of the zodiacal signs' (e.g. HI 18,23–4); more fully, ὁ λόξος καὶ διὰ μέσων τῶν ζῳδίων κύκλος, 'the inclined circle through the middle of the signs' (HI 64,4). Occasionally, when the context is clear, simply λόξος κύκλος, 'inclined circle' (HI 8,22). However, the latter can be used for other things, notably the moon's orbit (which is 'inclined' to the ecliptic). I normally use 'ecliptic' throughout.

[zodiacal] sign. The conventional subdivision of the ecliptic into twelve 30° stretches named Aries, Taurus, etc. For this Ptolemy uses, not ζῴδιον ('animal sign'), but δωδεκατημόριον ('twelfth'), presumably because he wishes to

distinguish the ecliptic, a notional circle, from the zodiac, a band of actual constellations.

star. The Greek term ἀστήρ really means 'heavenly body', and can be used indifferently for a star (in the modern sense), a planet, or even the sun and moon. When Ptolemy wishes to distinguish what we call stars, he says 'fixed stars'. I have normally translated ἀστήρ according to the context, as 'planet', 'star' or 'body'. However, in I 3-8, where Ptolemy uses the term to include all heavenly bodies, I too have used *star* in this special sense. When naming the five planets, Ptolemy almost always uses the periphrasis 'star of . . ', thus ὁ τοῦ Κρόνου [ἀστήρ], '[star] of Kronos'. I always translate simply 'Saturn' etc.

latitude (*celestial*). πλάτος (literally 'breadth') refers not only to 'the direction orthogonal to the ecliptic', but to any 'vertical' direction, e.g. that normal to the equator. In such cases I use, not 'latitude', but another appropriate term (see I 12 p. 63 with n. 74). In VII 3, however, I have been forced to use 'latitude' to express the more general meaning of the Greek (see p. 329 n.55).

Ptolemy uses ἔκκεντρος as both adjective and noun. It may be that in the latter case one has always to understand ἔκκεντρος κύκλος, 'eccentric circle'. However, to avoid ambiguity, I have (following mediaeval usage) consistently denoted the noun by *eccentre* and the adjective by *eccentric*. An 'eccentre' is simply an eccentric circle. Similarly for *concentre* and *concentric*.

I have occasionally used the convenient mediaeval term *deferent* to denote the circle on which an epicycle is 'carried'. Ptolemy has no one-word equivalent, but uses phrases like 'the concentric carrying the epicycle', 'the circle carrying it'.

anomaly. As noted e.g. by Pedersen (139 with n.9), ἀνωμαλία in the Almagest has a number of different meanings. Despite the ambiguity, I have generally rendered ἀνωμαλία and the adjective from which it is derived, ἀνώμαλος, by 'anomaly', 'anomalistic', although where necessary I have translated the latter literally as 'non-uniform'. Besides referring to non-uniform motion, 'anomaly' is also used for the mean (hence uniform) motion of the moon and planets on their epicycles (because the motion on the epicycle produces the appearance of 'non-uniformity'). For the planets Ptolemy distinguishes between the *synodic anomaly* (ἡ πρὸς τὸν ἥλιον ἀνωμαλία, 'the anomaly with respect to the sun', HII 255,8), which produces the phenomena of retrogradation and varies with the planet's elongation from the sun, and the *ecliptic anomaly* (ζῳδιακὴ ἀνωμαλία, HII 258,11), which varies according to the planet's position in the ecliptic.

equation. I use this convenient mediaeval term for the angle (or arc) to be applied to a mean motion to 'correct' it to account for a particular feature of the geometric model. Ptolemy uses the vaguer terms τὸ διάφορον 'difference' (which is also used for many other things) and προσθαφαίρεσις ('amount to be added or subtracted'). *equation of anomaly* refers to the correction for the varying position of a body on its epicycle, and *equation of centre* (only in the footnotes, not the text) to the correction due to the eccentricity of a planet's deferent.

centrum. I have occasionally used this mediaeval term in the footnotes to denote the angular distance from apogee (see below) to the centre of the epicycle.

elongation (ἀποχή) is the angular distance along the ecliptic between two bodies or points. It is used particularly, but not exclusively, for the ecliptic distance between sun and moon.

apogee and *perigee* are simply transcriptions of ἀπόγειον and περίγειον, literally '[point] far from earth' and '[point] near to earth'. These are the usual terms for the points on a body's orbit which are respectively farthest from and nearest to the terrestrial observer. Ptolemy also uses the superlative forms ἀπογειότατον (περιγειότατον) σημεῖον ('point farthest from (nearest to) earth'), with no obvious difference in meaning. However, in the case of Mercury, translation of both by 'perigee' generates an ambiguity. For all other bodies, in Ptolemy's models, the perigee is diametrically opposite the apogee, but for Mercury the point of closest approach is about 120° from apogee. Ptolemy still refers to the point 180° from apogee as the 'perigee' (περιγεῖον) for Mercury, and when he wants to refer to the point of that planet's closest approach uses the superlative (περιγειότατος). I have mitigated the ambiguity by translating the latter, not as 'perigee', but as 'closest to earth' (for Mercury alone).

phase. Used for the fixed stars and planets, this is simply a transcription of φάσις, and is a general term including all the significant 'configurations with respect to the sun' (listed by Ptolemy at VIII 4 pp. 409–10, and exemplified in his partially extant work φάσεις ἀπλανῶν ἀστέρων, 'Phases of the Fixed Stars'), such as first visibility at sunset, or last visibility just before dawn. But the literal meaning of φάσις is 'appearance', and Ptolemy also uses it to mean specifically 'first visibility' of a body after a period of invisibility. To avoid ambiguity, I have translated the latter case by 'first visibility', reserving 'phase' for the general term.

(iv) *Referring to sun and moon*

conjunction is a fairly literal rendering of σύνοδος ('meeting'), but *opposition* renders πανσέληνος (literally 'full moon', which occurs when sun and moon are in opposition). *syzygy* is a transcription of the convenient συζυγία (literally 'yoking together'), a general term to denote either or both conjunction and opposition. In eclipses the partial phases are denoted by *immersion* (ἔμπτωσις, 'falling in', the phase from the beginning of the eclipse to totality) and *emersion* (ἀναπλήρωσις, 'filling up again', the phase from the end of totality to the end of the eclipse). The total phase is denoted by μονή ('remaining') and rendered by *duration (of totality)*.

(v) *Time-reckoning*

Ptolemy often uses the term νυχθήμερον, which combines the Greek words for night and day, to mean the 'solar day' of 24 hours. There is no such convenient term in English. I have generally translated it *day* when no ambiguity is possible, but have occasionally resorted to periphrasis (e.g. II 3 p. 79 = HI 96, 7-9). Since we use clocks, we reckon time by the *mean solar day* of uniform length, the average time taken by the sun to go from one meridian crossing to the next. In antiquity, where the normal means of telling time was the sundial, it was usually reckoned by the *true solar day*, of varying length, the time taken by the sun to go from one meridian crossing to the next on a specific day. In III 9 Ptolemy explains why they are different, and how to transform one into the other. He uses the terms ὁμαλὰ νυχθήμερα ('uniform days') and ἀνώμαλα νυχθήμερα ('non-uniform days') for mean and true solar days respectively. When he is talking about intervals, he often refers to those measured in true solar days as 'reckoned simply', and those measured in mean solar days as 'reckoned accurately'.

The kind of hours normally used in the ancient world were *seasonal hours* (ὧραι καιρικαί), sometimes known as 'civil hours'. An hour was $\frac{1}{12}$th of the actual length of daylight or night-time at a given place, and hence the length of an hour varied according to terrestrial latitude and time of year, and a day-hour was of different length from a night-hour except at equinox. For astronomical purposes, however, the uniform $\frac{1}{24}$th of a day was used; these were known as *equinoctial hours* (ὧραι ἰσημεριναί), because they were the same length as the seasonal hour at equinox. If an ordinal number is attached to an hour, it indicates a seasonal hour, counted from dawn (or sunset, if specified by 'of night' or by the context). Thus 'the sixth hour' is the same as noon.

time-degrees. Another way of measuring time was by the amount of the celestial equator which had passed a bound (horizon or meridian). This was often connected with the rising-times of ecliptic arcs (see pp. 18-19). This measurement was in degrees. Since 360° of the equator cross the meridian in about one day, one 'time-degree' equals $\frac{1}{15}$th of an equinoctial hour or 4 minutes. The Greek term is χρόνοι ἰσημερινοί ('equatorial times'), sometimes abbreviated to χρόνοι ('times').

(vi) *Other*

mean (μέσος) can imply 'of average length' (as in 'mean synodic month') or 'uniform' (as in 'mean motion in longitude').

hypothesis. With some hesitation, I have used this to translate ὑπόθεσις, although the connotation in the Almagest never really coincides with the modern one. Whereas we use 'hypothesis' to denote a tentative theory which has still to be verified, Ptolemy usually means by ὑπόθεσις something more like 'model', 'system of explanation', often indeed referring to 'the hypotheses which we have demonstrated'. The word still retains much of the etymological meaning of 'basis on which something else is constructed'. The corresponding verbal forms are ὑποτίθεται, ὑποκεῖται, which I have frequently translated, not only as 'assume', but even as 'it is given'. They are standard terms of Greek geometry in this sense at least as early as Euclid.

6. *Editorial procedures*

Since the translation is based principally on the Teubner text of Heiberg (see p. 3), it is keyed to that edition by the addition of Heiberg's page numbers in the margin. There and elsewhere references to Heiberg are preceded by 'H'. Thus HI 236,15 means 'Heiberg's edition, Vol. I p. 236 line 15'. Where the context makes it unnecessary the volume number is omitted.

Brackets are used as follows. Square brackets [ ] enclose explanatory additions to or expansions of the Greek text by the translator. Curved brackets { } enclose passages which I believe to be later additions to Ptolemy's original text. Parentheses ( ) are used merely for clarity, better to express the author's sequence of thought.

As explained on p. 5, I believe the list of chapter headings preceding each book to be a later addition. Nevertheless, since these serve a useful purpose, I have grouped them together at the beginning (pp. 27–32) to serve as a table of contents.

I have made no effort to provide a continuous commentary, but refer the reader to the relevant sections in Olaf Pedersen's *A Survey of the Almagest* (abbreviated 'Pedersen') and O. Neugebauer's *A History of Ancient Mathematical Astronomy* (abbreviated *HAMA*). My footnotes are confined to particulars not treated by them, or requiring some elaboration, and to textual corrections. In Appendix A, however, I have provided worked examples of every type of problem (including, where it is not utterly trivial, the use of the tables) which arises in the Almagest, except where Ptolemy himself gives a worked example. Where possible, my example is taken from a calculation or observation actually occurring in the Almagest. Appendix B lists all my corrections to Heiberg's text. Appendix C discusses the problem of the derivation of Ptolemy's planetary mean motions, which has never been adequately treated.

The index includes all proper names occurring in the text, and certain selected topics (mostly taken from the Introduction and footnotes). It also contains all observations recorded in the Almagest, under the topic or body concerned (e.g. 'equinox', 'moon'). For a list of the observations in chronological order the reader is referred to Pedersen's Appendix A.

In drawing the diagrams I have tried to reproduce the manuscript tradition, while at the same time making the figures as clear as possible by marking the points unambiguously. Since there is often considerable variation in the manuscript representations, I have been forced to make many choices; but I have not 'modernized' the figures. Where a figure seemed inadequate, I have not changed it, but have added an explanatory one of my own. Such explanatory (and other supplementary) figures are distinguished by alphabetical numbering ('Fig. A' etc.), whereas figures reproduced from the manuscripts are numbered according to the book and the order within that book (thus 'Fig. 3.10' indicates that this is the tenth diagram in Book III; in the manuscripts they are not usually numbered, but where they are, they are numbered separately in each book). I have represented the Greek letters of the figures by the following system:

Text	Trans.	Text	Trans.	Text	Trans.
Α	A	Ι	J	Π	P
Β	B	Κ	K	Ρ	R
Γ	G	Λ	L	Σ	S
Δ	D	Μ	M	Τ	T
Ε	E	Ν	N	Υ	Y
Ζ	Z	Ξ	X	Φ	F
Η	H	Ο	O	Χ	Q
Θ	Θ			Ψ	V

7. *Other conventional symbols and abbreviations*

e	eccentricity
r	radius of epicycle or body
M	length of longest day in hours
m	length of shortest day in hours
R	radius of principal circle (e.g. of deferent)
α	(1) right ascension (see p. 18)
	(2) anomaly (see p. 21)
β	celestial latitude
δ	declination
ε	obliquity of ecliptic
η	elongation
θ	equation
ι	inclination of orbit (of moon or planet)
κ	'centrum', i.e. distance from apogee (see p. 22)
λ	longitude
ρ	(1) oblique ascension (see p. 18)
	(2) geocentric distance
φ	terrestrial latitude
ω	distance from northpoint on orbit

A bar over a letter denotes 'mean', thus $\bar{\lambda}$ = 'mean longitude'.

The following are used in a raised position (e.g. 2^p) to denote units:

d	days
h	equinoctial hours

m months
y years
p 'parts', i.e. the arbitrary units in trigonometrical calculations (see pp. 7–9)
° degrees
°° demi degrees ($2^{\circ\circ} = 1^\circ$, see p. 8)
°/d degrees per day

In the star catalogue only, * indicates some doubt about the reading. For other abbreviations particular to the star catalogue see p. 341 n.95.

Zodiacal signs

♈ Aries	♈ 0° =	0°	in longitude
♉ Taurus	♉ 0° =	30°	
♊ Gemini	♊ 0° =	60°	
♋ Cancer	♋ 0° =	90°	
♌ Leo	♌ 0° =	120°	
♍ Virgo	♍ 0° =	150°	
♎ Libra	♎ 0° =	180°	
♏ Scorpius	♏ 0° =	210°	
♐ Sagittarius	♐ 0° =	240°	
♑ Capricornus	♑ 0° =	270°	
♒ Aquarius	♒ 0° =	300°	
♓ Pisces	♓ 0° =	330°	

Planetary symbols

♄ Saturn
♃ Jupiter
♂ Mars
♀ Venus
☿ Mercury
☉ Sun
☽ Moon

Other astronomical symbols

⊕ Earth
☊ ascending node
☋ descending node

On 'sexagesimal' representations such as 6,13;10,0,58 see pp. 6–7.

For the mathematical symbols || and ||| (both meaning 'is similar to') and ≡ ('is congruent to') see p. 17.

For 'M. T. I' and 'M. T. II' see p. 18.

For manuscript abbreviations see pp. 3–4.

Contents of the *Almagest*[1]

[1] These lists of the chapter headings are found in the mss. at the beginning of each book preceded by the words 'The following are the contents of Book *n* of Ptolemy's mathematical treatise'. I believe that the division into chapters and the chapter headings are later additions (see Introduction p. 5).

BOOK III

BOOK IV

[2] Most mss., followed by Heiberg, read at H86,20 κανονίων τῶν κατὰ δεκαμοιρίαν παράλληλον, which is untranslatable. I read with D κανονίων τῶν κατὰ παράλληλον. Someone who compared the text at II 8 (H 134,I), κανόνιον τῶν κατὰ δεκαμοιρίαν ἀναφορῶν, imported δεκαμοιρίαν here and tried to combine the two inconsistent descriptions.

BOOK V

BOOK VI

[3] In the text the 'method of calculation' is explained at the end of ch. 9, and ch. 10 consists solely of the table. This variation is perhaps a remnant of a different chapter division. Cf. Introduction p. 5.

BOOK VII

BOOK VIII

BOOK IX

page

BOOK X

BOOK XI

BOOK XII

BOOK XIII

Translation of the ALMAGEST

Book I

1. {*Preface*}[4]

The true philosophers, Syrus,[5] were, I think, quite right to distinguish the theoretical part of philosophy from the practical. For even if practical philosophy, before it *is* practical, turns out to be theoretical,[6] nevertheless one can see that there is a great difference between the two: in the first place, it is possible for many people to possess some of the moral virtues even without being taught, whereas it is impossible to achieve theoretical understanding of the universe without instruction; furthermore, one derives most benefit in the first case [practical philosophy] from continuous practice in actual affairs, but in the other [theoretical philosophy] from making progress in the theory. Hence we thought it fitting to guide our actions (under the impulse of our actual ideas [of H5
what is to be done]) in such a way as never to forget, even in ordinary affairs, to strive for a noble and disciplined disposition, but to devote most of our time to intellectual matters, in order to teach theories, which are so many and beautiful, and especially those to which the epithet 'mathematical' is particularly applied. For Aristotle divides theoretical philosophy too, very fittingly, into three primary categories, physics, mathematics and theology.[7] For everything that exists is composed of matter, form and motion; none of these [three] can be observed in its substratum by itself, without the others: they can only be imagined. Now the first cause of the first motion of the universe, if one considers it simply, can be thought of as an invisible and motionless deity; the division [of theoretical philosophy] concerned with investigating this [can be called] 'theology', since this kind of activity, somewhere up in the highest reaches of the universe, can only be imagined, and is completely separated from

[4] This 'philosophical' preface and its relationship to Ptolemy's attitude to philosophy is discussed by Boll, *Studien* 68–76, to which the reader is referred for the relevant passages in ancient literature. The general standpoint is Aristotelian.

[5] Syrus is also the addressee of a number of other works by Ptolemy (see Toomer[5] 187). Nothing is known about him. The name is very common in (but not confined to) Greco-Roman Egypt. The statement in a scholion to the Tetrabiblos (quoted by Boll, *Studien* 67, n. 2) that some say he was a fictitious person, others that he was a doctor, merely reveals that he was equally unknown in late antiquity.

[6] Theon in his commentary (Rome II 320, 13–14) gives φησὶ . . . συμβεβηκέναι τῷ πρακτικῷ τό πρότερον αὐτοῦ τοῦ θεωρητικοῦ τυγχάνειν. This is a paraphrase rather than a different reading, but shows that he understood the text as I have translated it. By this obscure expression I take Ptolemy to mean that before actually practising virtues one must have some concept of them (even though this is innate rather than taught).

[7] E. g. Metaphysics E 1, 1026a 18 ff., ὥστε τρεῖς ἂν εἶεν φιλοσοφίαι θεωρητικαί, μαθηματική, φυσική, θεολογική.

perceptible reality. The division [of theoretical philosophy] which investigates material and ever-moving nature, and which concerns itself with 'white', 'hot', 'sweet', 'soft' and suchlike qualities one may call 'physics'; such an order of being is situated (for the most part) amongst corruptible bodies and below the lunar sphere. That division [of theoretical philosophy] which determines the
H6 nature involved in forms and motion from place to place, and which serves to investigate shape, number, size, and place, time and suchlike, one may define as 'mathematics'. Its subject-matter falls as it were in the middle between the other two, since, firstly, it can be conceived of both with and without the aid of the senses, and, secondly, it is an attribute of all existing things without exception, both mortal and immortal: for those things which are perpetually changing in their inseparable form, it changes with them, while for eternal things which have an aethereal[8] nature, it keeps their unchanging form unchanged.

From all this we concluded:[9] that the first two divisions of theoretical philosophy should rather be called guesswork than knowledge, theology because of its completely invisible and ungraspable nature, physics because of the unstable and unclear nature of matter; hence there is no hope that philosophers will ever be agreed about them; and that only mathematics can provide sure and unshakeable knowledge to its devotees, provided one approaches it rigorously. For its kind of proof proceeds by indisputable methods, namely arithmetic and geometry. Hence we were drawn to the investigation of that part of theoretical philosophy, as far as we were able to the whole of it, but especially to the theory concerning divine and heavenly things. For that alone is devoted to the investigation of the eternally
H7 unchanging. For that reason it too can be eternal and unchanging (which is a proper attribute of knowledge) in its own domain, which is neither unclear nor disorderly. Furthermore it can work in the domains of the other [two divisions of theoretical philosophy] no less than they do. For this is the best science to help theology along its way, since it is the only one which can make a good guess at [the nature of] that activity which is unmoved and separated; [it can do this because] it is familiar with the attributes of those beings[10] which are on the one hand perceptible, moving and being moved, but on the other hand eternal and unchanging, [I mean the attributes] having to do with motions and the arrangements of motions. As for physics, mathematics can make a significant contribution. For almost every peculiar attribute of material nature becomes apparent from the peculiarities of its motion from place to place. [Thus one can distinguish] the corruptible from the incorruptible by [whether it undergoes] motion in a straight line or in a circle, and heavy from light, and passive from active, by [whether it moves] towards the centre or away from the centre. With

[8] 'aethereal' (αἰθερώδης) has a precise meaning in Aristotelian physics: everything above the sphere of the moon is composed of an 'incorruptible' substance, unlike anything known on earth in its consistency (very thin) and in its natural motion (circular). See I 3 p. 40. One of the names for this substance is 'aether', another 'fifth essence'. See Campanus IV n. 56, pp. 394–5.

[9] In this exaltation of mathematics above the other two divisions of philosophy Ptolemy parts company with Aristotle, for whom theology was the most noble pursuit for the human mind.

[10] The heavenly bodies.

regard to virtuous conduct in practical actions and character, this science, above all things, could make men see clearly; from the constancy, order, symmetry and calm which are associated with the divine, it makes its followers lovers of this divine beauty, accustoming them and reforming their natures, as it were, to a similar spiritual state.

It is this love of the contemplation of the eternal and unchanging which we constantly strive to increase, by studying those parts of these sciences which H8 have already been mastered by those who approached them in a genuine spirit of enquiry, and by ourselves attempting to contribute as much advancement as has been made possible by the additional time between those people and ourselves.[11] We shall try to note down[12] everything which we think we have discovered up to the present time; we shall do this as concisely as possible and in a manner which can be followed by those who have already made some progress in the field.[13] For the sake of completeness in our treatment we shall set out everything useful for the theory of the heavens in the proper order, but to avoid undue length we shall merely recount what has been adequately established by the ancients. However, those topics which have not been dealt with [by our predecessors] at all, or not as usefully as they might have been, will be discussed at length, to the best of our ability.

2. {*On the order of the theorems*}

In the treatise which we propose, then, the first order of business is to grasp the relationship of the earth taken as a whole to the heavens taken as a whole.[14] In the treatment of the individual aspects which follows, we must first discuss the position of the ecliptic[15] and the regions of our part of the inhabited world and also the features differentiating each from the others due to the [varying] latitude at each horizon taken in order.[16] For if the theory of these matters is H9 treated first it will make examination of the rest easier. Secondly, we have to go through the motion of the sun and of the moon, and the phenomena accompanying these [motions];[17] for it would be impossible to examine the theory of the stars[18] thoroughly without first having a grasp of these matters. Our final task in this way of approach is the theory of the stars. Here too it would be appropriate to deal first with the sphere of the so-called 'fixed stars',[19]

[11] This notion of the advancement of science, and particularly astronomy, by the additional time available is one to which Ptolemy recurs in the epilogue (XIII 11 p. 647), and also, in a specifically astronomical context, at VII 1 p. 321 and VII 3 p. 329.

[12] ὑπομνηματίσασθαι. A ὑπόμνημα is a 'memoir', usually implying summary brevity. Ptolemy recurs to this too in the epilogue (XIII 11 p. 647).

[13] Ptolemy assumes that his readers will have a certain competence. See Introduction p. 6.

[14] I 3–8. On the logic of Ptolemy's order see Introduction pp. 5–6.

[15] I 12–16. The mathematical section I 10–11 is not specifically mentioned here.

[16] Book II.

[17] Books III–VI.

[18] 'Stars' here and throughout chs. 3–8 includes both fixed stars and planets (see Introduction p. 21) and also, sometimes, sun and moon.

[19] Books VII–VIII.

and follow that by treating the five 'planets', as they are called.[20] We shall try to provide proofs in all of these topics by using as starting-points and foundations, as it were, for our search the obvious phenomena, and those observations made by the ancients and in our own times which are reliable. We shall attach the subsequent structure of ideas to this [foundation] by means of proofs using geometrical methods.

The general preliminary discussion covers the following topics: the heaven is spherical in shape, and moves as a sphere; the earth too is sensibly spherical in shape, when taken as a whole; in position it lies in the middle of the heavens very much like its centre; in size and distance it has the ratio of a point to the sphere of
H10 the fixed stars; and it has no motion from place to place. We shall briefly discuss each of these points for the sake of reminder.

3. {*That the heavens move like a sphere*}[21]

It is plausible to suppose that the ancients got their first notions on these topics from the following kind of observations. They saw that the sun, moon and other stars were carried from east to west along circles which were always parallel to each other, that they began to rise up from below the earth itself, as it were, gradually got up high, then kept on going round in similar fashion and getting lower, until, falling to earth, so to speak, they vanished completely, then, after remaining invisible for some time, again rose afresh and set; and [they saw] that the periods of these [motions], and also the places of rising and setting, were, on the whole, fixed and the same.

What chiefly led them to the concept of a sphere was the revolution of the ever-visible stars, which was observed to be circular, and always taking place about one centre, the same [for all]. For by necessity that point became [for
H11 them] the pole of the heavenly sphere: those stars which were closer to it revolved on smaller circles, those that were farther away described circles ever greater in proportion to their distance, until one reaches the distance of the stars which become invisible. In the case of these, too, they saw that those near the ever-visible stars remained invisible for a short time, while those farther away remained invisible for a long time, again in proportion [to their distance]. The result was that in the beginning they got to the aforementioned notion solely from such considerations; but from then on, in their subsequent investigation, they found that everything else accorded with it, since absolutely all phenomena are in contradiction to the alternative notions which have been propounded.

For if one were to suppose that the stars' motion takes place in a straight line towards infinity, as some people have thought,[22] what device could one

[20] Books IX–XIII.

[21] See Pedersen 36–7.

[22] According to Theon's commentary (Rome II 338) this belief was Epicurean, but I know of no other evidence. The only other relevant passage appears to be Xenophanes, Diels-Kranz A41a (the sun really moves towards infinity).

conceive of which would cause each of them to appear to begin their motion from the same starting-point every day? How could the stars turn back if their motion is towards infinity? Of, if they did turn back, how could this not be obvious? [On such a hypothesis], they must gradually diminish in size until they disappear, whereas, on the contrary, they are seen to be greater at the very moment of their disappearance, at which time they are gradually obstructed and cut off, as it were, by the earth's surface.

But to suppose that they are kindled as they rise out of the earth and are extinguished again as they fall to earth is a completely absurd hypothesis.[23] For even if we were to concede that the strict order in their size and number, their H12
intervals, positions and periods could be restored by such a random and chance process; that one whole area of the earth has a kindling nature, and another an extinguishing one, or rather that the same part [of the earth] kindles for one set of observers and extinguishes for another set; and that the same stars are already kindled or extinguished for some observers while they are not yet for others: even if, I say, we were to concede all these ridiculous consequences, what could we say about the ever-visible stars, which neither rise nor set? Those stars which are kindled and extinguished ought to rise and set for observers everywhere, while those which are not kindled and extinguished ought always to be visible for observers everywhere. What cause could we assign for the fact that this is not so? We will surely not say that stars which are kindled and extinguished for some observers never undergo this process for other observers. Yet it is utterly obvious that the same stars rise and set in certain regions [of the earth] and do neither at others.

To sum up, if one assumes any motion whatever, except spherical, for the heavenly bodies, it necessarily follows that their distances, measured from the earth upwards, must vary, wherever and however one supposes the earth itself to be situated. Hence the sizes and mutual distances of the stars must appear to vary for the same observers during the course of each revolution, since at one H13
time they must be at a greater distance, at another at a lesser. Yet we see that no such variation occurs. For the apparent increase in their sizes at the horizons[24] is caused, not by a decrease in their distances, but by the exhalations of moisture surrounding the earth being interposed between the place from which we observe and the heavenly bodies, just as objects placed in water appear bigger than they are, and the lower they sink, the bigger they appear.

The following considerations also lead us to the concept of the sphericity of the heavens. No other hypothesis but this can explain how sundial constructions produce correct results; furthermore, the motion of the heavenly bodies is the most unhampered and free of all motions, and freest motion belongs among

[23] Theon (Rome II 340) ascribes this to Heraclitus. Otherwise it is attested for Xenophanes (Diels-Kranz A38), and was admitted as one possible explanation by Epicurus (e.g. *Letter to Pythocles* 92) and his followers.

[24] Ptolemy refers to the well-known phenomenon that the sun and moon appear larger when close to the horizon. He gives an incorrect physical and optical explanation here. In a later work (*Optics* III 60, ed. Lejeune p. 116) he correctly explains it as a purely psychological phenomenon. No doubt instrumental measurement of the apparent diameters had convinced him that the enlargement is entirely illusory.

plane figures to the circle and among solid shapes to the sphere; similarly, since of different shapes having an equal boundary those with more angles are greater [in area or volume], the circle is greater than [all other] surfaces, and the sphere greater than [all other] solids;[25] [likewise] the heavens are greater than all other bodies.

Furthermore, one can reach this kind of notion from certain physical
H14 considerations. E.g., the aether is, of all bodies, the one with constituent parts which are finest and most like each other; now bodies with parts like each other have surfaces with parts like each other; but the only surfaces with parts like each other are the circular, among planes, and the spherical, among three-dimensional surfaces. And since the aether is not plane, but three-dimensional, it follows that it is spherical in shape. Similarly, nature formed all earthly and corruptible bodies out of shapes which are round but of unlike parts, but all aethereal and divine bodies out of shapes which are of like parts and spherical. For if they were flat or shaped like a discus[26] they would not always display a circular shape to all those observing them simultaneously from different places on earth. For this reason it is plausible that the aether surrounding them, too, being of the same nature, is spherical, and because of the likeness of its parts moves in a circular and uniform fashion.

4. {*That the earth too, taken as a whole, is sensibly spherical*}[27]

That the earth, too, taken as a whole,[28] is sensibly spherical can best be grasped from the following considerations. We can see, again, that the sun, moon and
H15 other stars do not rise and set simultaneously for everyone on earth, but do so earlier for those more towards the east, later for those towards the west. For we find that the phenomena at eclipses, especially lunar eclipses,[29] which take place at the same time [for all observers], are nevertheless not recorded as occurring at the same hour (that is at an equal distance from noon) by all observers. Rather, the hour recorded by the more easterly observers is always later than that recorded by the more westerly. We find that the differences in the hour are proportional to the distances between the places [of observation]. Hence one can reasonably conclude that the earth's surface is spherical, because its evenly curving surface (for so it is when considered as a whole) cuts off [the heavenly bodies] for each set of observers in turn in a regular fashion.

If the earth's shape were any other, this would not happen, as one can see from the following arguments. If it were concave, the stars would be seen rising first by those more towards the west; if it were plane, they would rise and set

[25] These propositions were proved in a work by Zenodorus (early second century B.C., see Toomer[1]) from which extensive excerpts are given by (among others) Theon (Rome II 355–79). There is a good summary in Heath *HGM* II 207–13.

[26] The only relevant passage I know is Empedocles, Diels-Kranz A60, who maintained that the moon is disk-shaped.

[27] See Pedersen 37–9.

[28] 'taken as a whole': ignoring local irregularities such as mountains, which are negligible in comparison to the total mass.

[29] The timings for solar eclipses are complicated by parallax.

simultaneously for everyone on earth; if it were triangular or square or any other polygonal shape, by a similar argument, they would rise and set simultaneously for all those living on the same plane surface. Yet it is apparent that nothing like this takes place. Nor could it be cylindrical, with the curved surface in the east-west direction, and the flat sides towards the poles of the universe, H16
which some might suppose more plausible. This is clear from the following: for those living on the curved surface none of the stars would be ever-visible, but either all stars would rise and set for all observers, or the same stars, for an equal [celestial] distance from each of the poles, would always be invisible for all observers. In fact, the further we travel toward the north, the more[30] of the southern stars disappear and the more of the northern stars appear. Hence it is clear that here too the curvature of the earth cuts off [the heavenly bodies] in a regular fashion in a north-south direction, and proves the sphericity [of the earth] in all directions.

There is the further consideration that if we sail towards mountains or elevated places from and to any direction whatever, they are observed to increase gradually in size as if rising up from the sea itself in which they had previously been submerged: this is due to the curvature of the surface of the water.

5. {*That the earth is in the middle of the heavens*}[31]

Once one has grasped this, if one next considers the position of the earth, one will find that the phenomena associated with it could take place only if we H17
assume that it is in the middle of the heavens, like the centre of a sphere. For if this were not the case, the earth would have to be either

[a] not on the axis [of the universe] but equidistant from both poles, or
[b] on the axis but removed towards one of the poles, or
[c] neither on the axis nor equidistant from both poles.

Against the first of these three positions militate the following arguments. If we imagined [the earth] removed towards the zenith or the nadir of some observer, then, if he were at *sphaera recta*, he would never experience equinox, since the horizon would always divide the heavens into two unequal parts, one above and one below the earth; if he were at *sphaera obliqua*, either, again, equinox would never occur at all, or, [if it did occur,] it would not be at a position halfway between summer and winter solstices, since these intervals would necessarily be unequal, because the equator, which is the greatest of all parallel circles drawn about the poles of the [daily] motion, would no longer be bisected by the horizon; instead [the horizon would bisect] one of the circles parallel to the equator, either to the north or to the south of it. Yet absolutely everyone agrees that these intervals are equal everywhere on earth, since H18
[everywhere] the increment of the longest day over the equinoctial day at the

[30] Reading πλείονα (with D) for τὰ πλείονα at H16,9. Corrected by Manitius.
[31] See Pedersen 39–42.

summer solstice is equal to the decrement of the shortest day from the equinoctial day at the winter solstice. But if, on the other hand, we imagined the displacement to be towards the east or west of some observer, he would find that the sizes and distances of the stars would not remain constant and unchanged at eastern and western horizons, and that the time-interval from rising to culmination would not be equal to the interval from culmination to setting. This is obviously completely in disaccord with the phenomena.

Against the second position, in which the earth is imagined to lie on the axis removed towards one of the poles, one can make the following objections. If this were so, the plane of the horizon would divide the heavens into a part above the earth and a part below the earth which are unequal and always different for different latitudes,[32] whether one considers the relationship of the same part at two different latitudes or the two parts at the same latitude. Only at *sphaera recta* could the horizon bisect the sphere; at a *sphaera obliqua* situation such that the nearer pole were the ever-visible one, the horizon would always make the part above the earth lesser and the part below the earth greater; hence another phenomenon would be that the great circle of the ecliptic would be divided into
H19 unequal parts by the plane of the horizon. Yet it is apparent that this is by no means so. Instead, six zodiacal signs are visible above the earth at all times and places, while the remaining six are invisible; then again [at a later time] the latter are visible in their entirety above the earth, while at the same time the others are not visible. Hence it is obvious that the horizon bisects the zodiac, since the same semi-circles are cut off by it, so as to appear at one time completely above the earth, and at another [completely] below it.

And in general, if the earth were not situated exactly below the [celestial] equator, but were removed towards the north or south in the direction of one of the poles, the result would be that at the equinoxes the shadow of the gnomon at sunrise would no longer form a straight line with its shadow at sunset in a plane parallel to the horizon, not even sensibly.[33] Yet this is a phenomenon which is plainly observed everywhere.

It is immediately clear that the third position enumerated is likewise impossible, since the sorts of objection which we made to the first [two] will both arise in that case.

To sum up, if the earth did not lie in the middle [of the universe], the whole order of things which we observe in the increase and decrease of the length of daylight would be fundamentally upset. Furthermore, eclipses of the moon would not be restricted to situations where the moon is diametrically opposite the sun (whatever part of the heaven [the luminaries are in]), since the earth
H20 would often come between them when they were not diametrically opposite, but at intervals of less than a semi-circle.

[32] The word translated here and elsewhere as '[terrestrial] latitude' is κλίμα, for the meaning of which see Introduction p. 19.

[33] The *caveat* 'sensibly' is inserted because the equinox is not a date but an instant of time. Therefore on the day of equinox the sun does not rise due east and set due west (as is implied by the rising and setting shadows lying on the same straight line). However, the difference would be 'imperceptible to the senses'.

6. {*That the earth has the ratio of a point to the heavens*}[34]

Moreover, the earth has, to the senses, the ratio of a point to the distance of the sphere of the so-called fixed stars.[35] A strong indication of this is the fact that the sizes and distances of the stars, at any given time, appear equal and the same from all parts of the earth everywhere, as observations of the same [celestial] objects from different latitudes are found to have not the least discrepancy from each other. One must also consider the fact that gnomons set up in any part of the earth whatever, and likewise the centres of armillary spheres,[36] operate like the real centre of the earth; that is, the lines of sight [to heavenly bodies] and the paths of shadows caused by them agree as closely with the [mathematical] hypotheses explaining the phenomena as if they actually passed through the real centre-point of the earth.

Another clear indication that this is so is that the planes drawn through the observer's lines of sight at any point [on earth], which we call 'horizons', always bisect the whole heavenly sphere. This would not happen if the earth were of H21
perceptible size in relation to the distance of the heavenly bodies; in that case only the plane drawn through the centre of the earth could bisect the sphere, while a plane through any point on the surface of the earth would always make the section [of the heavens] below the earth greater than the section above it.

7. {*That the earth does not have any motion from place to place, either*}[37]

One can show by the same arguments as the preceding that the earth cannot have any motion in the aforementioned directions, or indeed ever move at all from its position at the centre. For the same phenomena would result as would if it had any position other than the central one. Hence I think it is idle to seek for causes for the motion of objects towards the centre, once it has been so clearly established from the actual phenomena that the earth occupies the middle place in the universe, and that all heavy objects are carried towards the earth. The following fact alone would most readily lead one to this notion [that all objects fall towards the centre]. In absolutely all parts of the earth, which, as we said, has been shown to be spherical and in the middle of the universe, the direction[38] and path of the motion (I mean the proper, [natural] motion) of all H22
bodies possessing weight is always and everywhere at right angles to the rigid plane drawn tangent to the point of impact. It is clear from this fact that, if

[34] See Pedersen 42–3.

[35] Ptolemy qualifies the traditional terminology for the fixed stars as 'so-called' (καλουμένων) because they do in fact, according to him, have a motion (the modern 'precession'). He develops the point further at VII 1 p. 321, q.v. In general, however, he uses the traditional terminology without qualification.

[36] An example of an armillary sphere (κρικωτὴ σφαῖρα) is the 'astrolabe' described in V 1. For references to the term in other works see LSJ s.v. κρικωτός.

[37] See Pedersen 43–4.

[38] πρόσνευσις, which I have translated 'the direction of motion' here, means basically 'direction in which something points' (for astronomical usages see V 5 p. 227 n. 19 and VI 11 p. 313 n. 77). Thus it would also include here the direction of a plumb-line (cf. I 12 p. 62).

[these falling objects] were not arrested by the surface of the earth, they would certainly reach the centre of the earth itself, since the straight line to the centre is also always at right angles to the plane tangent to the sphere at the point of intersection [of that radius] and the tangent.

Those who think it paradoxical that the earth, having such a great weight, is not supported by anything and yet does not move, seem to me to be making the mistake of judging on the basis of their own experience instead of taking into account the peculiar nature of the universe. They would not, I think, consider such a thing strange once they realised that this great bulk of the earth, when compared with the whole surrounding mass [of the universe], has the ratio of a point to it. For when one looks at it in that way, it will seem quite possible that that which is relatively smallest should be overpowered and pressed in equally from all directions to a position of equilibrium by that which is the greatest of all
H23 and of uniform nature. For there is no up and down in the universe with respect to itself,[39] any more than one could imagine such a thing in a sphere: instead the proper and natural motion of the compound bodies in it is as follows: light and rarefied bodies drift outwards towards the circumference, but seem to move in the direction which is 'up' for each observer, since the overhead direction for all of us, which is also called 'up', points towards the surrounding surface;[40] heavy and dense bodies, on the other hand, are carried towards the middle and the centre, but seem to fall downwards, because, again, the direction which is for all us towards our feet, called 'down', also points towards the centre of the earth. These heavy bodies, as one would expect, settle about the centre because of their mutual pressure and resistance, which is equal and uniform from all directions. Hence, too, one can see that it is plausible that the earth, since its total mass is so great compared with the bodies which fall towards it, can remain motionless under the impact of these very small weights (for they strike it from all sides), and receive, as it were, the objects falling on it. If the earth had a single motion in common with other heavy objects, it is obvious that it would be carried down faster than all of them because of its much greater size: living
H24 things and individual heavy objects would be left behind, riding on the air, and the earth itself would very soon have fallen completely out of the heavens. But such things are utterly ridiculous merely to think of.

But certain people,[41] [propounding] what they consider a more persuasive view, agree with the above, since they have no argument to bring against it, but think that there could be no evidence to oppose their view if, for instance, they supposed the heavens to remain motionless, and the earth to revolve from west to east about the same axis [as the heavens], making approximately one revolution each day;[42] or if they made both heaven and earth move by any amount whatever, provided, as we said, it is about the same axis, and in such a

[39] Reading αὐτόν (with D, Is) for αὐτήν at H23,1.

[40] It is not clear to me whether Ptolemy means the outmost boundary of the *universe* or merely the surface (of the 'aether') surrounding the *earth*.

[41] Heraclides of Pontos (late fourth century B.C.) is the earliest certain authority for the view that the earth rotates on its axis. See *HAMA* II 694–6. It was also adopted by Aristarchus as part of his more radical heliocentric hypothesis.

[42] 'approximately' because one revolution takes place in a sidereal, not a solar day.

way as to preserve the overtaking of one by the other. However, they do not realise that, although there is perhaps nothing in the celestial phenomena which would count against that hypothesis, at least from simpler considerations, nevertheless from what would occur here on earth and in the air, one can see that such a notion is quite ridiculous. Let us concede to them [for the sake of argument] that such an unnatural thing could happen as that the most rare and light of matter should either not move at all or should move in a way no different from that of matter with the opposite nature (although things in the air, which are less rare [than the heavens] so obviously move with a more rapid motion than any earthy object); [let us concede that] the densest and heaviest objects H25
have a proper motion of the quick and uniform kind which they suppose (although, again, as all agree, earthy objects are sometimes not readily moved even by an external force). Nevertheless, they would have to admit that the revolving motion of the earth must be the most violent of all motions associated with it, seeing that it makes one revolution in such a short time; the result would be that all objects not actually standing on the earth would appear to have the same motion, opposite to that of the earth: neither clouds nor other flying or thrown objects would ever be seen moving towards the east, since the earth's motion towards the east would always outrun and overtake them, so that all other objects would seem to move in the direction of the west and the rear. But if they said that the air is carried around in the same direction and with the same speed as the earth, the compound objects in the air would none the less always seem to be left behind by the motion of both [earth and air]; or if those objects too were carried around, fused, as it were, to the air, then they would never appear to have any motion either in advance or rearwards: they would always appear still, neither wandering about nor changing position, whether they were flying or thrown objects. Yet we quite plainly see that they do undergo all these H26
kinds of motion, in such a way that they are not even slowed down or speeded up at all by any motion of the earth.

8. {*That there are two different primary motions in the heavens*}[43]

It was necessary to treat the above hypotheses first as an introduction to the discussion of particular topics and what follows after. The above summary outline of them will suffice, since they will be completely confirmed and further proven by the agreement with the phenomena of the theories which we shall demonstrate in the following sections. In addition to these hypotheses, it is proper, as a further preliminary, to introduce the following general notion, that there are two different primary motions in the heavens. One of them is that which carries everything from east to west: it rotates them with an unchanging and uniform motion along circles parallel to each other, described, as is obvious, about the poles of this sphere which rotates everything uniformly. The greatest of these circles is called the 'equator',[44] because it is the only [such

[43] See Pedersen 45.
[44] 'equator': ἰσημερινός, literally 'of equal day' or 'equinoctial'. See Introduction p. 19.

parallel circle] which is always bisected by the horizon (which is a great circle), and because the revolution which the sun makes when located on it produces equinox everywhere, to the senses. The other motion is that by which the
H27 spheres of the stars perform movements in the opposite sense to the first motion, about another pair of poles, which are different from those of the first rotation. We suppose that this is so because of the following considerations. When we observe for the space of any given single day, all heavenly objects whatever are seen, as far as the senses can determine, to rise, culminate and set at places which are analogous and lie on circles parallel to the equator; this is characteristic of the first motion. But when we observe continuously without interruption over an interval of time, it is apparent that while the other stars retain their mutual distances and (for a long time) the particular characteristics arising from the positions they occupy as a result of the first motion,[45] the sun, the moon and the planets have certain special motions which are indeed complicated and different from each other, but are all, to characterise their general direction,[46] towards the east and opposite to [the motion of] those stars which preserve their mutual distances and are, as it were, revolving on one sphere.

Now if this motion of the planets too took place along circles parallel to the equator, that is, about the poles which produce the first kind of revolution, it
H28 would be sufficient to assign a single kind of revolution to all alike, analogous to the first. For in that case it would have seemed plausible that the movements which they undergo are caused by various retardations, and not by a motion in the opposite direction. But as it is, in addition to their movement towards the east, they are seen to deviate continuously to the north and south [of the equator]. Moreover the amount of this deviation cannot be explained as the result of a uniformly-acting force pushing them to the side: from that point of view it is irregular, but it is regular if considered as the result of [motion on] a circle inclined to the equator. Hence we get the concept of such a circle, which is one and the same for all planets, and particular to them. It is precisely defined and, so to speak, drawn by the motion of the sun, but it is also travelled by the moon and the planets, which always move in its vicinity, and do not randomly pass outside a zone on either side of it which is determined for each body. Now since this too is shown to be a great circle, since the sun goes to the north and south of the equator by an equal amount, and since, as we said, the eastward motion of all of the planets takes place on one and the same circle, it became necessary to suppose that this second, different motion of the whole takes place
H29 about the poles of the inclined circle we have defined [i.e. the ecliptic], in the opposite direction to the first motion.

If, then, we imagine a great circle drawn through the poles of both the above-mentioned circles, (which will necessarily bisect each of them, that is the equator and the circle inclined to it [the ecliptic], at right angles), we will have four points on the ecliptic: two will be produced by [the intersection of] the

[45] These characteristics of the fixed stars are e.g. dates of first and last visibility. They are unchanged 'for a long time' because the effect of precession is very slow.

[46] The qualification is inserted here to allow for the retrogradations of the planets.

equator, diametrically opposite each other; these are called 'equinoctial' points. The one at which the motion [of the planets] is from south to north is called the 'spring' equinox, the other the 'autumnal'. Two [other points] will be produced by [the intersection of] the circle drawn through both poles; these too, obviously, will be diametrically opposite each other; they are called 'tropical' [or 'solsticial'] points. The one south of the equator is called the 'winter' [solstice], the one north, the 'summer' [solstice].

We can imagine the first primary motion, which encompasses all the other motions, as described and as it were defined by the great circle drawn through both poles [of equator and ecliptic] revolving, and carrying everything else with it, from east to west about the poles of the equator. These poles are fixed, so to speak, on the 'meridian' circle, which differs from the aforementioned [great] H30
circle in the single respect that it is not drawn through the poles of the ecliptic too at all positions of the latter. Moreover, it is called 'meridian' because it is considered to be always orthogonal to the horizon.[47] For a circle in such a position divides both hemispheres, that above the earth and that below it, into two equal parts, and defines the midpoint of both day and night.

The second, multiple-part motion is encompassed by the first and encompasses the spheres of all the planets. As we said, it is carried around by the aforementioned [first motion], but itself goes in the opposite direction about the poles of the ecliptic, which are also fixed on the circle which produces the first motion, namely the circle through both poles [of ecliptic and equator]. Naturally they [the poles of the ecliptic] are carried around with it [the circle through both poles], and, throughout the period of the second motion in the opposite direction, they always keep the great circle of the ecliptic, which is described by that [second] motion, in the same position with respect to the equator.[48]

9. {*On the individual concepts*}

Such, then are the necessary preliminary concepts which must be summarily set out in our general introduction. We are now about to begin the individual demonstrations, the first of which, we think, should be to determine the size of H31
the arc between the aforementioned poles [of the ecliptic and equator] along the great circle drawn through them. But we see that it is first necessary to explain the method of determining chords:[49] we shall demonstrate the whole topic geometrically once and for all.

[47] See Introduction p.19.

[48] My translation follows the interpretation of Theon (Rome II 447). Manitius (p. 24 n. a) wrongly considers τοῦ γραφομένου δι' αὐτῆς μεγίστου κὰι λοξοῦ κύκλου interpolated, partly because he misinterprets συντηροῦσιν (which is used here in a way similar to συντηροῦσαν at HI 6,10).

[49] 'chords': literally 'straight lines in a circle'. On this term see Introduction p. 17.

Book III

{*Preface*}[1]

In the preceding part of our treatise we have dealt with those aspects of heaven and earth which required, in outline, a preliminary mathematical discussion; H191
also the inclination of the sun's path through the ecliptic, and the resultant particular phenomena, both at *sphaera recta* and at *sphaera obliqua* for every inhabited region. We think that we should [now] discuss, as the subject which appropriately follows the above, the theory of the sun and moon, and go through the phenomena which are a consequence of their motions. For none of the phenomena associated with the [other] heavenly bodies can be completely investigated without the previous treatment of these [two]. Furthermore, we find that the subject of the sun's motion must take first place amongst these [sun and moon], since without that it would, again, be impossible to give a complete discussion of the moon's theory from start to finish.

1. {*On the length of the year*}[2]

The very first of the theorems concerning the sun is the determination of the length of the year. The ancients were in disagreement and confusion in their pronouncements on this topic, as can be seen from their treatises, especially those of Hipparchus, who was both industrious and a lover of truth. The main cause of the confusion on this topic which even he displayed is the fact that, when one examines the apparent returns [of the sun] to [the same] equinox or solstice, one finds that the length of the year exceeds 365 days by less than $\frac{1}{4}$-day, but when H192
one examines its return to [one of] the fixed stars it is greater [than $365\frac{1}{4}$ days]. Hence Hipparchus comes to the idea that the sphere of the fixed stars too has a very slow motion, which, just like that of the planets, is towards the rear with respect to the revolution producing the first [daily] motion, which is that of a [great] circle drawn through the poles of both equator and ecliptic.[3]

As for us, we shall show this is indeed the case, and how it takes place, in our discussion of the fixed stars[4] (the theory of the fixed stars, too, cannot be

[1] D and part of the Arabic tradition (L, P, but not Q, T) begin chapter 1 at this point. On such variations, and the conclusion to be drawn, see Introduction p. 5.

[2] See *HAMA* 54–5, Pedersen 128–34.

[3] This characterisation of the daily motion by means of the rotation of a great circle through the poles of equator and ecliptic refers back to I 8 p. 47.

[4] VII 2–3.

yearly motions above and the hourly motions below, and the third will contain the monthly motions above and the daily motions below. The numbers representing time will be in the first [i.e. left-hand] section, and the corresponding degrees, obtained by successive addition of the appropriate amount for each [time-unit], in the second [i.e. right-hand] section. The tables are as follows.

2. {*Table of the mean motion of the sun*}[29] H210—15

[See pp. 142-3.]

3. {*On the hypotheses for uniform circular motion*}[30] H216

Our next task is to demonstrate the apparent anomaly of the sun. But first we must make the general point that the rearward displacements of the planets with respect to the heavens are, in every case, just like the motion of the universe in advance, by nature uniform and circular. That is to say, if we imagine the bodies or their circles being carried around by straight lines, in absolutely every case the straight line in question describes equal angles at the centre of its revolution in equal times. The apparent irregularity [anomaly] in their motions is the result of the position and order of those circles in the sphere of each by means of which they carry out their movements, and in reality there is in essence nothing alien to their eternal nature in the 'disorder' which the phenomena are supposed to exhibit. The reason for the appearance of irregularity can be explained by two hypotheses, which are the most basic and simple. When their motion is viewed with respect to a circle imagined to be in the plane of the ecliptic, the centre of which coincides with the centre of the universe (thus its centre can be considered to coincide with our point of view), then we can suppose, either that the uniform motion of each [body] takes place on a circle which is not concentric with the universe, or that they have such a concentric circle, but their uniform motion takes place, not actually on that circle, but on H217
another circle, which is carried by the first circle, and [hence] is known as the 'epicycle'. It will be shown that either of these hypotheses will enable [the planets] to appear, to our eyes, to traverse unequal arcs of the ecliptic (which is concentric to the universe) in equal times.

In the eccentric hypothesis: [see Fig. 3.1] we imagine the eccentric circle, on which the body travels with uniform motion, to be ABGD on centre E, with diameter AED, on which point Z represents the observer.[31] Thus A is the apogee, and D the perigee. We cut off equal arcs AB and DG, and join BE, BZ, GE and GZ. Then it is immediately obvious that the body will traverse the arcs

[29] Corrections to Heiberg's text: H210, 23–5, column of fourths (for arguments 342, 360 and 378). A misprint has disrupted the order, which should be λ, να, ιβ, but has become να, ιβ, λ (51, 12, 30). H215,38, thirds : λε (35): λϛ, as Is.

[30] See *HAMA* 55–7, Pedersen 134–44.

[31] 'the observer'; literally 'our point of view'.

TABLE OF THE SUN'S MEAN MOTION

Distance [in Anomaly] from the Sun's Apogee in ♊ 5;30° to its Mean Longitude in the 1st Year of Nabonassar, ♓ 0;45° : 265;15°

18-Year Periods	°	′	″	‴	⁗	′′′′′	′′′′′′
18	355	37	25	36	20	34	30
36	351	14	51	12	41	9	0
54	346	52	16	49	1	43	30
72	342	29	42	25	22	18	0
90	338	7	8	1	42	52	30
108	333	44	33	38	3	27	0
126	329	21	59	14	24	1	30
144	324	59	24	50	44	36	0
162	320	36	50	27	5	10	30
180	316	14	16	3	25	45	0
198	311	51	41	39	46	19	30
216	307	29	7	16	6	54	0
234	303	6	32	52	27	28	30
252	298	43	58	28	48	3	0
270	294	21	24	5	8	37	30
288	289	58	49	41	29	12	0
306	285	36	15	17	49	46	30
324	281	13	40	54	10	21	0
342	276	51	6	30	30	55	30
360	272	28	32	6	51	30	0
378	268	5	57	43	12	4	30
396	263	43	23	19	32	39	0
414	259	20	48	55	53	13	30
432	254	58	14	32	13	48	0
450	250	35	40	8	34	22	30
468	246	13	5	44	54	57	0
486	241	50	31	21	15	31	30
504	237	27	56	57	36	6	0
522	233	5	22	33	56	40	30
540	228	42	48	10	17	15	0
558	224	20	13	46	37	49	30
576	219	57	39	22	58	24	0
594	215	35	4	59	18	58	30
612	211	12	30	35	39	33	0
630	206	49	56	12	0	7	30
648	202	27	21	48	20	42	0
666	198	4	47	24	41	16	30
684	193	42	13	1	1	51	0
702	189	19	38	37	22	25	30
720	184	57	4	13	43	0	0
738	180	34	29	50	3	34	30
756	176	11	55	26	24	9	0
774	171	49	21	2	44	43	30
792	167	26	46	39	5	18	0
810	163	4	12	15	25	52	30

Single Years	°	′	″	‴	⁗	′′′′′	′′′′′′
1	359	45	24	45	21	8	35
2	359	30	49	30	42	17	10
3	359	16	14	16	3	25	45
4	359	1	39	1	24	34	20
5	358	47	3	46	45	42	55
6	358	32	28	32	6	51	30
7	358	17	53	17	28	0	5
8	358	3	18	2	49	8	40
9	357	48	42	48	10	17	15
10	357	34	7	33	31	25	50
11	357	19	32	18	52	34	25
12	357	4	57	4	13	43	0
13	356	50	21	49	34	51	35
14	356	35	46	34	56	0	10
15	356	21	11	20	17	8	45
16	356	6	36	5	38	17	20
17	355	52	0	50	59	25	55
18	355	37	25	36	20	34	30

Hours	°	′	″	‴	⁗	′′′′′	′′′′′′
1	0	2	27	50	43	3	1
2	0	4	55	41	26	6	2
3	0	7	23	32	9	9	3
4	0	9	51	22	52	12	5
5	0	12	19	13	35	15	6
6	0	14	47	4	18	18	7
7	0	17	14	55	1	21	9
8	0	19	42	45	44	24	10
9	0	22	10	36	27	27	11
10	0	24	38	27	10	30	12
11	0	27	6	17	53	33	14
12	0	29	34	8	36	36	15
13	0	32	1	59	19	39	16
14	0	34	29	50	2	42	18
15	0	36	57	40	45	45	19
16	0	39	25	31	28	48	20
17	0	41	53	22	11	51	21
18	0	44	21	12	54	54	23
19	0	46	49	3	37	57	24
20	0	49	16	54	21	0	25
21	0	51	44	45	4	3	27
22	0	54	12	35	47	6	28
23	0	56	40	26	30	9	29
24	0	59	8	17	13	12	31

Months	°	′	″	‴	⁗	′′′′′	′′′′′′
30	29	34	8	36	36	15	30
60	59	8	17	13	12	31	0
90	88	42	25	49	48	46	30
120	118	16	34	26	25	2	0
150	147	50	43	3	1	17	30
180	177	24	51	39	37	33	0
210	206	59	0	16	13	48	30
240	236	33	8	52	50	4	0
270	266	7	17	29	26	19	30
300	295	41	26	6	2	35	0
330	325	15	34	42	38	50	30
360	354	49	43	19	15	6	0

Days	°	′	″	‴	⁗	′′′′′	′′′′′′
1	0	59	8	17	13	12	31
2	1	58	16	34	26	25	2
3	2	57	24	51	39	37	33
4	3	56	33	8	52	50	4
5	4	55	41	26	6	2	35
6	5	54	49	43	19	15	6
7	6	53	58	0	32	27	37
8	7	53	6	17	45	40	8
9	8	52	14	34	58	52	39
10	9	51	22	52	12	5	10
11	10	50	31	9	25	17	41
12	11	49	39	26	38	30	12
13	12	48	47	43	51	42	43
14	13	47	56	1	4	55	14
15	14	47	4	18	18	7	45
16	15	46	12	35	31	20	16
17	16	45	20	52	44	32	47
18	17	44	29	9	57	45	18
19	18	43	37	27	10	57	49
20	19	42	45	44	24	10	20
21	20	41	54	1	37	22	51
22	21	41	2	18	50	35	22
23	22	40	10	36	3	47	53
24	23	39	18	53	17	0	24
25	24	38	27	10	30	12	55
26	25	37	35	27	43	25	26
27	26	36	43	44	56	37	57
28	27	35	52	2	9	50	28
29	28	35	0	19	23	2	59
30	29	34	8	36	36	15	30

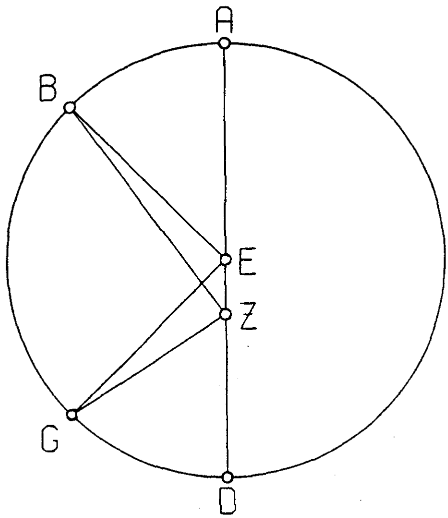

Fig. 3. 1

AB and GD in equal times, but will [in so doing] appear to have traversed unequal arcs of a circle drawn on centre Z. For

$$\angle \text{BEA} = \angle \text{GED}.$$
$$\text{But } \angle \text{BZA} < \angle \text{BEA (or } \angle \text{GED)},$$
$$\text{and } \angle \text{GZD} > \angle \text{GED (or } \angle \text{BEA)}.$$

H218 In the epicyclic hypothesis: we imagine [see Fig. 3.2] the circle concentric with the ecliptic as ABGD on centre E, with diameter AEG, and the epicycle carried by it, on which the body moves, as ZHΘK on centre A.

Then here too it is immediately obvious that, as the epicycle traverses circle ABGD with uniform motion, say from A towards B, and as the body traverses the epicycle with uniform motion, then when the body is at points Z and Θ, it will appear to coincide with A, the centre of the epicycle, but when it is at other points it will not. Thus when it is, e.g., at H, its motion will appear greater than the uniform motion [of the epicycle] by arc AH, and similarly when it is at K its motion will appear less than the uniform by arc AK.

Now in this kind of eccentric hypothesis[32] the least speed always occurs at the apogee and the greatest at the perigee, since ∠ AZB [in Fig. 3.1] is always less than ∠ DZG. But in the epicyclic hypothesis both this and the reverse are possible. For the motion of the epicycle is towards the rear with respect to the
H219 heavens, say from A towards B [in Fig. 3.2]. Now if the motion of the body on the epicycle is such that it too moves rearwards from the apogee, that is from Z towards H, the greatest speed will occur at the apogee, since at that point both

[32] Ptolemy is hinting at the existence of another kind of eccentric hypothesis, one which is geometrically equivalent to that epicyclic hypothesis in which the sense of rotation is the same for both planet and epicycle. But he does not discuss this until XII 1 (p. 555), where we learn that the equivalence was already known to Apollonius of Perge (*c.* 200 B.C.). See *HAMA* 149–50.

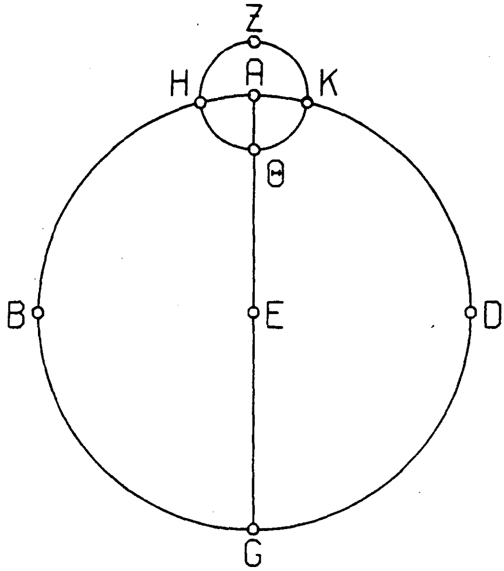

Fig. 3.2

epicycle and body are moving in the same direction. But if the motion of the body from the apogee is in advance on the epicycle, that is from Z towards K, then the reverse will occur: the least speed will occur at the apogee, since at that point the body is moving in the opposite direction to the epicycle.

Having established that, we must next make the additional preliminary point that for bodies which exhibit a double anomaly both the above hypotheses may be combined, as we shall prove in our discussions of such bodies, but for a body which displays a single invariant anomaly, a single one of the above hypotheses will suffice; and [in this case] all the phenomena will be represented, with no difference, by either hypothesis, provided that the same ratios are preserved in both. By this I mean that the ratio, in the eccentric hypothesis, of the distance between the centre of vision and the centre of the eccentre to the radius of the eccentre, must be the same as the ratio, in the epicyclic hypothesis, of the radius of the epicycle to the radius of the deferent;[33] and furthermore that the time taken by the body, travelling towards the rear, to H220
traverse the immovable eccentre, must be the same as the time taken by the epicycle, also travelling towards the rear, to traverse the circle with the observer as centre [the deferent], while the body moves with equal [angular] speed about the epicycle, but so that its motion at the apogee [of the epicycle] is in advance.

If these conditions are fulfilled, the identical phenomena will result from either hypothesis. We shall briefly show this [now] by comparing the ratios in abstract, and later by means of the actual numbers we shall assign to them for

[33] 'deferent': see Introduction p. 21.

the sun's anomaly.[34] I say then, first, that in both hypotheses, the greatest difference between the uniform motion and the apparent, non-uniform motion (which is also the notional position of the mean speed for the bodies)[35] occurs when the apparent distance from the apogee comprises a quadrant, and that the time between apogee [position] and the above-mentioned mean speed [position] is greater than the time between mean speed and perigee. Hence, for the eccentric hypothesis always, and for the epicyclic hypothesis when the motion at apogee is in advance, the time from least speed to mean is greater
H221 than the time from mean speed to greatest; for in both hypotheses the slowest motion takes place at the apogee. But [for the epicyclic hypothesis] when the sense of revolution of the body is rearwards from the apogee on the epicycle, the reverse is true: the time from greatest speed to mean is greater than the time from mean to least, since in this case the greatest speed occurs at the apogee.

First, then, [see Fig. 3.3] let the body's eccenter be ABGD on centre E, with diameter AEG. On this diameter take the centre of the ecliptic, that is, the position of the observer, at Z, and draw BZD through Z at right angles to AEG. Let the positions of the body be B and D, so that, obviously, its apparent distance from apogee A is a quadrant on either side. We have to prove that the greatest difference between mean and anomalistic motion takes place at points B and D.

Join EB and ED.

It is immediately obvious that the ratio of ∠ EBZ to 4 right angles equals the

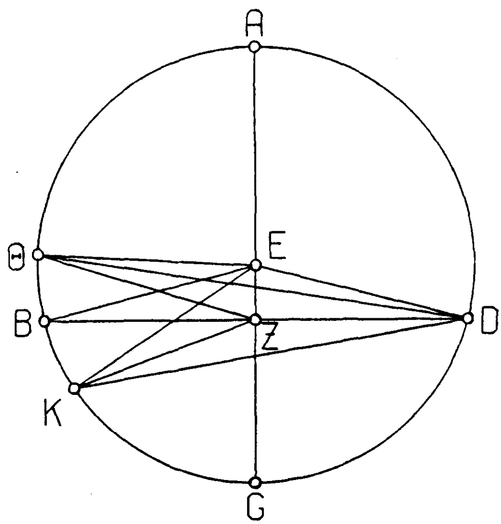

Fig. 3.3

[34] Reference to III 4 p. 157.

[35] Ptolemy never attempts to prove this statement about the position where the apparent motion equals the mean motion, but it is intuitively seen to be true from the epicyclic model. See *HAMA* 57, Pedersen 143.

ratio of the arc of the difference due to the anomaly[36] to the whole circle; for ∠ AEB subtends the arc of the uniform motion, and ∠ AZB subtends the arc of the apparent, non-uniform motion, and the difference between them is ∠ EBZ. H222

I say, then, that no angle greater than these two [∠ EBZ and ∠ EDZ] can be constructed on line EZ at the circumference of circle ABGD.

[Proof:] Construct at points Θ and K angles EΘZ and EKZ, and join ΘD, KD. Then since, in any triangle, the greater side subtends the greater angle,[37]

and ΘZ > ZD,

∴ ∠ ΘDZ > ∠ DΘZ.

But ∠ EDΘ = ∠ EΘD, since EΘ = ED [radii].

Therefore, by addition, ∠ EDZ (=∠ EBD) > ∠ EΘZ.

Again, since DZ > KZ,

∠ ZKD > ∠ ZDK.

But ∠ EKD = ∠ EDK, since EK = ED.

Therefore, by subtraction, ∠ EDZ (= ∠ EBZ) > ∠ EKZ.

Therefore it is impossible for any other angle to be constructed in the way defined greater than those at points B and D. H223

Simultaneously it is proven that arc AB, which represents the time from least speed to mean, exceeds BG, which represents the time from mean speed to greatest, by twice the arc comprising the equation of anomaly. For ∠ AEB exceeds a right angle (∠ EZB) by ∠ EBZ, and ∠ BEG falls short of a right angle by the same amount.

To prove the same theorem again for the other hypothesis, let [Fig. 3.4] the circle concentric with the universe be ABG on centre D and diameter ADB, and let the epicycle which is carried around it in the same plane be EZH on centre A. Let us suppose the body to be at H when its apparent distance from the apogee is a quadrant. Join AH and DHG.

I say that DHG is tangent to the epicycle; for that is the position in which the difference between uniform and anomalistic motion is greatest. H224

[Proof:] The mean motion, counted from the apogee, is represented by ∠ EAH; for the body traverses the epicycle with the same [angular] speed as the epicycle traverses circle ABG. Furthermore the difference between mean and apparent motion is represented by ∠ ADH. Therefore it is clear that the amount by which ∠ EAH exceeds ∠ ADH (namely ∠ AHD) represents the apparent distance of the body from the apogee. But this distance is, by hypothesis, a quadrant. Therefore ∠ AHD is a right angle, and hence line DHG is tangent to epicycle EZH. Therefore arc AG, since it comprises the distance between the centre A and the tangent, is the greatest possible difference due to the anomaly.

By the same reasoning, arc EH, which according to the sense of rotation on

[36] This expression is later used as a technical term for the angle corresponding to ∠ EBZ here, and is usually translated 'equation of anomaly'. See Introduction pp. 21–2.

[37] Precisely this statement, that the greater angle is subtended by the greater side, is the enunciation of Euclid I 19 (which Heiberg refers to ad loc.). But in fact what underlies Ptolemy's statement is that, if side a is greater than side b, angle A is greater than angle B, which is Euclid I 18. Perhaps we should adopt the reading of D, ὑπὸ τὴν μείζονα πλευρὰν ἡ μείζων γωνία ὑποτείνει ('the greater angle subtends the greater side'), and assume that the text has been assimilated to the (wrong) Euclidean wording.

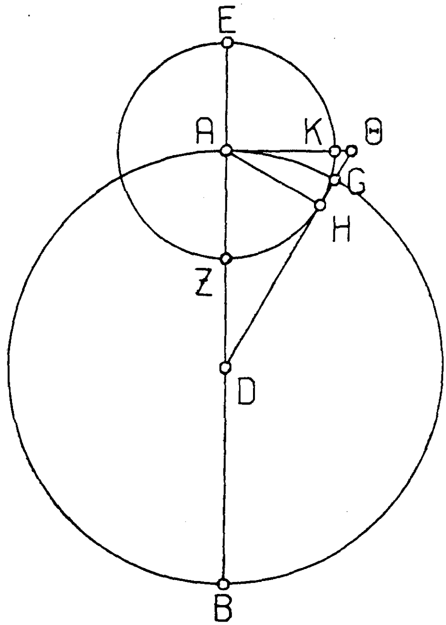

Fig. 3.4

the epicycle assumed here, represents the time from least speed to mean, exceeds arc HZ, which represents the time from mean speed to greatest, by
H225 twice arc AG. For if we produce DH to Θ and draw AKΘ at right angles to EZ,

$$\angle \text{ KAH} = \angle \text{ ADG},^{38}$$

$$\text{and arc KH} = \text{arc AG}.^{39}$$

And arc EKH is greater than a quadrant by arc KH,
while arc ZH is less than a quadrant by arc KH.

Q.E.D.

It is also true that the same effects will be produced by both hypotheses if one takes a partial motion over the same stretch of time for both, whether one considers the mean motion or the apparent, or the difference between them, that is the equation of anomaly. The best way to see that is as follows.

[See Fig. 3.5.][40] Let the circle concentric with the ecliptic be ABG on centre D, and let the circle which is eccentric but equal to the concentre ABG be EZH on centre Θ. Let the common diameter through their centres D, Θ and the

[38] Euclid VI 8.

[39] To get a grammatical text I excise ὁμοία at H225,4. It was introduced (at an early period, since it is reflected in the Arabic translations) as a correction of Ptolemy's inaccurate (to the scholastic mind) statement that arc KH *equals* arc AG. Since the arcs are on circles of different sizes, they are technically only 'similar'. An alternative correction would be ἴσαι μὲν γίγνονται αἵ τε ὑπὸ ΚΑΗ καὶ ΑΔΗ γωνίαι (which is actually found in Theon's commentary ad loc., Rome III 868,8, but is probably a paraphrase; it also seems to be behind L).

[40] The figure in Heiberg (p. 225) wrongly omits the letter corresponding to L (though this is found in all mss.). Manitius, misled by this, 'emended' ΑΛ at H226,23 to the nonsensical 'AB'.

apogee E be EAΘD. Cut off at random an arc AB on the concentre, and with centre B and radius DΘ draw the epicycle KZ. Join KBD.

I say that the body will be carried by both kinds of motion [i.e. according to both hypotheses] to point Z, the intersection of the eccentre and the epicycle, in the same time in all cases (that is, the three arcs, EZ on the eccentre, AB on the H226

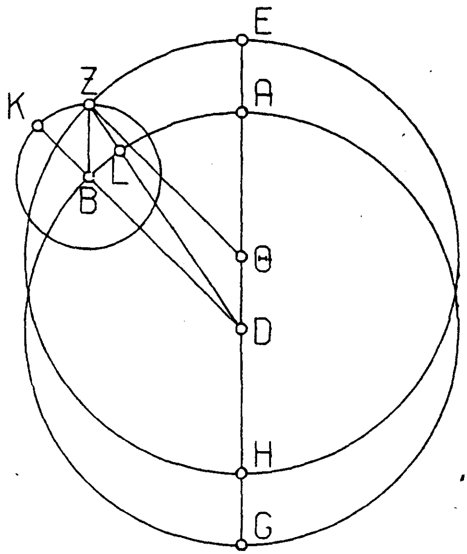

Fig. 3.5

concentre, and KZ on the epicycle, are all similar), and that the difference between uniform and anomalistic motion, and the apparent positions of the body, will turn out to be one and the same according to both hypotheses. [Proof:] Join ZΘ, BZ and DZ.

Since, in the quadrilateral BDΘZ, the opposite sides are equal, ZΘ to BD and BZ to DΘ, BDΘZ is a parallelogram.

Therefore $\angle$ EΘZ = $\angle$ ADB = $\angle$ ZBK.

Therefore, since they are angles at the centre [of circles], the arcs subtended by them are also similar, i.e.

Arc EZ of the eccentre || arc AB of the concentre || arc KZ of the epicycle.

Therefore the body will be carried by both kinds of motions in the same time to the same point, Z, and will appear to have traversed the same arc AL of the ecliptic from the apogee, and accordingly the equation of anomaly will be the same in both hypotheses; for we showed that that equation is represented by $\angle$ DZΘ in the eccentric hypothesis and by $\angle$ BDZ in the epicyclic hypothesis, and these two angles are alternate and equal, since, as we have shown, ZΘ is parallel to BD. H227

It is obvious that the same results will hold good for all distances [of the body from the apogee]. For quadrilateral ΘDZB will always be a parallelogram, and [hence] the motion of the body on the epicycle will actually describe the eccentric circle, provided the ratios[41] are similar and their members equal in both hypotheses.

Moreover, even if the members are unequal in size, provided their ratios are similar, the same phenomena will result. This can be shown as follows.

As before [see Fig. 3.6] let the circle concentric with the universe be ABG on centre D and the diameter, on which the body reaches apogee and perigee positions, ADG. Let the epicycle be drawn on point B, at an arbitrary distance, arc AB, from apogee A. Let the arc traversed by the body [on the epicycle] be
H228 EZ, which is, obviously, similar to AB, since the revolutions on [both] circles have the same period. Join DBE, BZ, DZ.

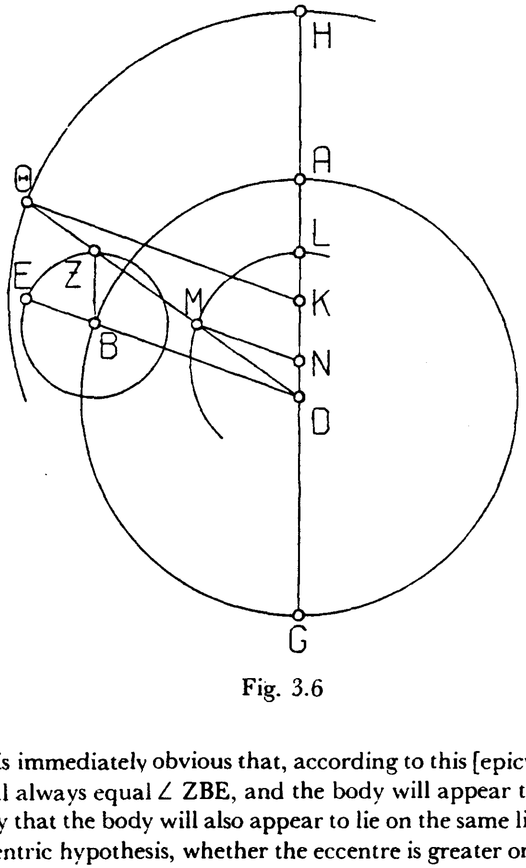

Fig. 3.6

Now it is immediately obvious that, according to this [epicyclic] hypothesis, ∠ ADE will always equal ∠ ZBE, and the body will appear to lie on line DZ.

But I say that the body will also appear to lie on the same line DZ according to the eccentric hypothesis, whether the eccentre is greater or smaller than the concentre ABG, provided only that one assumes that the ratios are similar and that the periods of revolution are the same.

[Proof:] Let the eccentre be drawn under the conditions we have described, greater [than the concentre] as HΘ on centre K ([which must lie] on AG), and

[41] The ratios are e:R and r:R.

smaller [than the concentre] as LM on centre N (this too [must lie on AG]). Produce DZ as DMZΘ, and DA as DLAH, and join ΘK, MN.

Then since

$$DB:BZ = \Theta K:KD = MN:ND \text{ [by hypothesis]},$$

and $\angle$ BZD = $\angle$ MDN (since DA is parallel to BZ); H229

the three triangles [ZDB,DΘK,DMN] are equiangular,

and $\angle$ BDZ = $\angle$ DΘK = $\angle$ DMN (angles subtended by corresponding sides).

Therefore DB, ΘK and MN are parallel.

$$\therefore \angle ADB = \angle AK\Theta = \angle ANM.$$

Since these angles are at the centres of their circles, the arcs on them, AB, HΘ and LM, will also be similar.

So it is true, not only that the epicycle has traversed arc AB in the same time as the body has traversed arc EZ, but also that the body will have traversed arcs HΘ and LM on the eccentres in that same time; hence in every case it will be seen along the same line DMZΘ, according to the epicyclic [hypothesis] at point Z, according to the greater eccentre at point Θ, and according to the smaller eccentre at point M. The same will hold true in all positions.

A further consequence is that where the apparent distance of the body from apogee [at one moment] equals its apparent distance from perigee [at another], the equation of anomaly will be the same at both positions.

[Proof:] In the eccentric hypothesis [see Fig. 3.7], we draw the eccentric circle

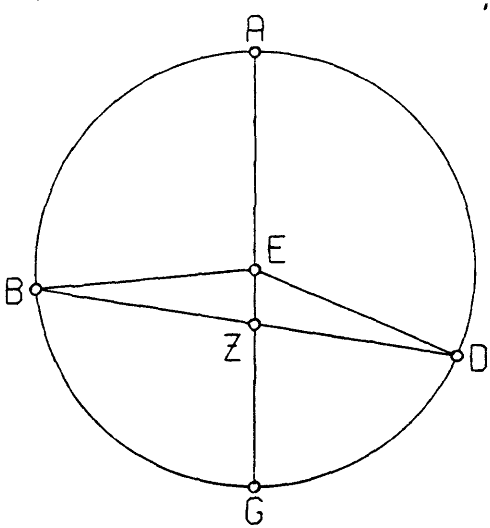

Fig. 3.7

ABGD on centre E and diameter AEG through apogee A. We suppose the observer to be located at Z, and draw an arbitrary [chord] BZD through Z, and join EB and ED. Then the apparent positions [of the body at B and D] will be equal and opposite, that is the angle AZB from the apogee will be equal and H230 opposite to angle GZD from the perigee; and the equation of anomaly will be the same [in both cases], since

$$BE = ED, \text{ and } \angle EBZ = \angle EDZ.$$

So the arc [AB] of mean motion counted from the apogee A will exceed the arc of apparent motion (i.e. the arc subtended by angle AZB) by the same equation [equal to $\angle$ EBZ] as the arc of mean motion counted from the perigee G is exceeded by the arc of apparent motion (i.e. the [equal] arc subtended by $\angle$ GZD). For

$$\angle AEB > \angle AZB, \text{ and } \angle GED < \angle GZD.$$

In the epicyclic hypothesis [see Fig. 3.8] if, as before, we draw the concentre ABG on centre D and diameter ADG, and the epicycle EZH on centre A, draw
H231 an arbitrary line DHBZ, and join AZ and AH, then the arc AB representing the equation of anomaly will be the same at both positions, i.e. whether the body is

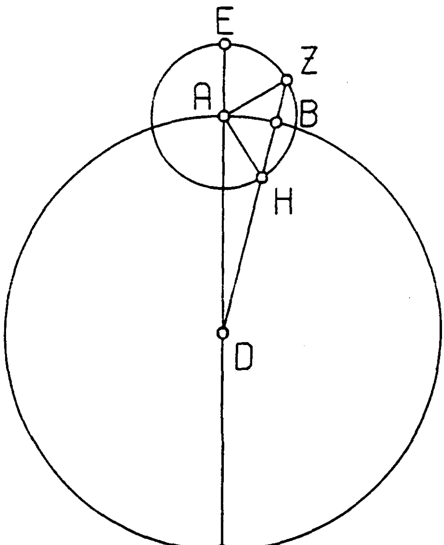

Fig. 3.8

at Z or at H. And the distance of the body from the point on the ecliptic corresponding to the apogee when it is at Z will be equal to its distance from the point corresponding to the perigee when it is at H. For the arc of its apparent distance from the apogee is represented by $\angle$ DZA, since, as we showed, this is the difference between the mean motion and the equation of anomaly.[42] And the arc of its apparent distance from the perigee is represented by $\angle$ ZHA (for this, too, is equal to the mean motion from the perigee plus the equation of anomaly).

$$\text{But } \angle DZA = \angle ZHA, \text{ since } AZ = AH.$$

[42] $\angle$ DZA = $\angle$ EAZ–$\angle$ ADZ. Shown p. 147.

Thus here too we conclude that the mean motion exceeds the apparent near the apogee (i.e. ∠ EAZ exceeds ∠ AZD) by the same equation (namely ∠ ADH) as the mean motion is exceeded by the (same) apparent motion (i.e. ∠ HAD by H232
∠ AHZ) near the perigee.

Q.E.D.

4. {*On the apparent anomaly of the sun*}[43]

Having set out the above preliminary theorems, we must add a further preliminary thesis concerning the apparent anomaly of the sun. This has to be a single anomaly, of such a kind that the time taken from least speed to mean shall always be greater than the time from mean speed to greatest, for we find that this accords with the phenomena. Now this could be represented by either of the hypotheses described above, though in case of the epicyclic hypothesis the motion of the sun on the apogee arc of the epicycle would have to be in advance. However, it would seem more reasonable to associate it with the eccentric hypothesis, since that is simpler and is performed by means of one motion instead of two.[44]

Our first task is to find the ratio of the eccentricity of the sun's circle, that is, the ratio which the distance between the centre of the eccentre and the centre of the ecliptic (located at the observer) bears to the radius of the eccentre. We must also find the degree of the ecliptic in which the apogee of the eccentre is located. These problems have been solved by Hipparchus with great care.[45] He assumes H233
that the interval from spring equinox to summer solstice is 94½ days, and that the interval from summer solstice to autumnal equinox is 92½ days, and then, with these observations as his sole data, shows that the line segment between the above-mentioned centres [of eccentre and ecliptic] is approximately $\frac{1}{24}$th of the radius of the eccentre, and that the apogee is approximately 24½° (where the ecliptic is divided into 360°) in advance of the summer solstice. We too, for our own time, find approximately the same values for the times [taken by the sun to traverse] the above-mentioned quadrants, and for those ratios. Hence it is clear to us that the sun's eccentre always maintains the same position relative to the solsticial and equinoctial points.[46]

In order not to neglect this topic, but rather to display the theorem worked out according to our own numerical solution, we too shall solve the problem, for the eccentre, using the same observed data, namely, as already stated, that the interval from spring equinox to summer solstice comprises 94½ days, and that

[43] See *HAMA* 57–8, Pedersen 144–9.

[44] On the desirability of simplicity in hypotheses see III 1 p. 136 with n.17.

[45] Reading μετὰ πάσης σπουδῆς (with D, Ar) at H233,1–2 for μετὰ σπουδῆς ('with care').

[46] According to Ptolemy the sun's apogee (unlike those of the five planets, as it later turns out, IX 7) does not share in the motion of precession. The reproaches that have been cast on Ptolemy (e.g. by Manitius I 428–9) for failing to discover that the sun's apogee too has a motion through the ecliptic are unjustified. To do that he would have needed observations of the time of equinox and solstice far more accurate than those available (to the nearest ¼-day), and not only for his own time but also for an earlier time. See the papers by Rome[3] and Petersen and Schmidt for a mathematical demonstration of this.

Book IX

1. {*On the order of the spheres of sun, moon and the 5 planets*}

Such, then, more or less, is the sum total of the chief topics one may mention as having to do with the fixed stars, in so far as the phenomena [observed] up to now provide the means of progress in our understanding. There remains, to [complete] our treatise, the treatment of the five planets. To avoid repetition we shall, as far as possible, explain the theory of the latter by means of an exposition common [to all five], treating each of the methods [for all planets] together.

First, then, [to discuss] the order of their spheres, which are all situated [with their poles] nearly coinciding with the poles of the inclined, ecliptic circle: we see that almost all the foremost astronomers agree that all the spheres are closer to the earth than that of the fixed stars, and farther from the earth than that of the moon, and that those of the three [outer planets] are farther from the earth than those of the other [two] and the sun, Saturn's being greatest, Jupiter's the next in order towards the earth, and Mars' below that. But concerning the H207
spheres of Venus and Mercury, we see that they are placed below the sun's by the more ancient astronomers, but by some of their successors these too are placed above [the sun's],[1] for the reason that the sun has never been obscured by them [Venus and Mercury] either. To us, however, such a criterion seems to have an element of uncertainty, since it is possible that some planets might indeed be below the sun, but nevertheless not always be in one of the planes through the sun and our viewpoint, but in another [plane], and hence might not be seen passing in front of it, just as in the case of the moon, when it passes below [the sun] at conjunction, no obscuration results in most cases.[2]

And since there is no other way, either, to make progress in our knowledge of this matter, since none of the stars[3] has a noticeable parallax (which is the only phenomenon from which the distances can be derived), the order assumed by the older [astronomers] appears the more plausible. For, by putting the sun in the middle, it is more in accordance with the nature [of the bodies] in thus

[1] There is a good deal of evidence for the identity of some of those who held the second opinion, including Plato, Eratosthenes and Archimedes. For details on this and other ancient arrangements see *HAMA* II 690-3.

[2] I.e. no transits of Venus or Mercury had been observed. Neugebauer has shown (*HAMA* 227-30) that transits are in fact predictable from Ptolemy's own theory. Ptolemy later seems to have realized this, for in the ***Planetary Hypotheses*** (ed. Goldstein 2,28,10-12) he says: 'if a body of such small size (as a planet) were to occult a body of such large size and with so much light (as the sun), it would necessarily be imperceptible, because of the smallness of the occulting body and the state of the parts of the sun's body which remain uncovered.' (Goldstein's translation here, p.6, is inaccurate).

[3] This includes both fixed stars and planets.

separating those which reach all possible distances from the sun and those which do not do so, but always move in its vicinity; provided only that it does not remove the latter close enough to the earth that there can result a parallax of any size.[4]

H208 2. {*On our purpose in the hypotheses of the planets*}

So much, then, for the arrangements of the spheres. Now it is our purpose to demonstrate for the five planets, just as we did for the sun and moon, that all their apparent anomalies can be represented by uniform circular motions, since these are proper to the nature of divine beings, while disorder and non-uniformity are alien [to such beings]. Then it is right that we should think success in such a purpose a great thing, and truly the proper end of the mathematical part of theoretical philosophy.[5] But, on many grounds, we must think that it is difficult, and that there is good reason why no-one before us has yet succeeded in it.[6] For, [firstly], in investigations of the periodic motions of a planet, the possible [inaccuracy] resulting from comparison of [two] observations (at each of which the observer may have committed a small observational error) will, when accumulated over a continuous period, produce a noticeable difference [from the true state] sooner when the interval [between the observations] over which the examination is made is shorter, and less soon when it is longer. But we have records of planetary observations only from a time which is recent in comparison with such a vast enterprise: this makes prediction for a time many times greater [than the interval for which observations are available] insecure. [Secondly], in investigation of the anomalies, considerable confusion stems from the fact that it is apparent that each planet exhibits two anomalies, which are moreover unequal both in their
H209 amount and in the periods of their return: one [return] is observed to be related to the sun, the other to the position in the ecliptic; but both anomalies are continuously combined, whence it is difficult to distinguish the characteristics of each individually. [It is] also [confusing] that most of the ancient [planetary] observations have been recorded in a way which is difficult to evaluate, and crude. For [1] the more continuous series of observations concern stations and phases [i.e. first and last visibilities].[7] But detection of both of these particular

[4] In his *Planetary Hypotheses* (see Goldstein's edition) Ptolemy proposes a system in which the spheres of the planets are contiguous; thus the greatest distance from the earth attained by a planet is equal to the least distance attained by the one next in order outwards. This appears to provide support for the order he adopts here, since it results in a solar distance very nearly the same as that obtained by a different method in *Almagest* V 15. Since this system also brings Mercury, at its least distance, to the moon's greatest distance (64 earth-radii), Mercury ought to exhibit a considerable parallax, contrary to what is enunciated here.

[5] Cf. I 1 p. 35.

[6] We cannot doubt that not only planetary theories but planetary tables had been constructed before Ptolemy: the proof is supplied by Indian astronomy, which is based on Greek theories which are largely, if not entirely, pre-Ptolemaic, and indeed by Ptolemy's own reference to the 'Aeon-tables' below (p. 422). What he means is that all previous efforts were, by his criteria, unsatisfactory

[7] Ptolemy is certainly thinking of the Babylonian planetary observations, which are characteristically of this type. They have become available to us through the 'diaries' (see Sachs[2]), but to Ptolemy were probably known only through Hipparchus' compilation (see p. 421).

phenomena is fraught with uncertainty: stations cannot be fixed at an exact moment, since the local motion of the planet for several days both before and after the actual station is too small to be observable; in the case of the phases, not only do the places [in which the planets are located] immediately become invisible together with the bodies which are undergoing their first or last visibility, but the times too can be in error, both because of atmospherical differences and because of differences in the [sharpness of] vision of the observers. [2] In general, observations [of planets] with respect to one of the fixed stars, when taken over a comparatively great distance, involve difficult computations and an element of guesswork in the quantity measured, unless one carries them out in a manner which is thoroughly competent and knowledgeable. This is not only because the lines joining the observed stars do not always form right angles with the ecliptic, but may form an angle of any size (hence one may expect considerable error in determining the position in latitude and longitude, due to H210
the varying inclination of the ecliptic [to the horizon frame of reference]); but also because the same interval [between star and planet] appears to the observer as greater near the horizon, and less near mid-heaven;[8] hence, obviously, the interval in question can be measured as at one time greater, at another less than it is in reality.

Hence it was, I think, that Hipparchus, being a great lover of truth, for all the above reasons, and especially because he did not yet have in his possession such a groundwork of resources in the form of accurate observations from earlier times as he himself has provided to us,[9] although he investigated the theories of the sun and moon, and, to the best of his ability, demonstrated with every means at his command that they are represented by uniform circular motions, did not even make a beginning in establishing theories for the five planets, not at least in his writings which have come down to us.[10] All that he did was to make a compilation of the planetary observations arranged in a more useful way,[11] and to show by means of these that the phenomena were not in agreement with the hypotheses of the astronomers of that time. For, we may presume, he thought that one must not only show that each planet has a twofold anomaly, or that each planet has retrograde arcs which are not constant, and are of such and such sizes (whereas the other astronomers had constructed their geometrical proofs on the basis of a single unvarying anomaly and retrograde arc); nor [that it was sufficient to show] that these anomalies can in fact be represented either

[8] This appears to be the only reference to the effect of refraction (if that is what it is) in the Almagest, despite its obvious relevance e.g. to the observations of Mercury's greatest elongations in IX 7. Ptolemy discusses it (as a theoretical problem) in some detail in *Optics* V 23–30 (ed. Lejeune 237–42).

[9] This seems to imply that Hipparchus recorded planetary observations of his own, which Ptolemy used to establish his theories. This may be true, but it is strange that Ptolemy cites not a single such observation by Hipparchus. Could Ptolemy mean merely that Hipparchus had not 'yet' assembled the compilation of earlier planetary observations which he mentions just below?

[10] The circulation of books in antiquity was so fortuitous that, even for one, like Ptolemy, who had access to the great resources of the libraries at Alexandria, this was a necessary *caveat*.

[11] I have little doubt that all the older planetary observations cited in the Almagest are derived from this compilation (cf. p. 452 n.66), and that part of Hipparchus' 'rearrangement' was to give their dates in the Egyptian calendar. For a similar service he rendered for the listing of lunar eclipses see *HAMA* 320–21.

H211 by means of eccentric circles or by circles concentric with the ecliptic, and carrying epicycles, or even by combining both, the ecliptic anomaly being of such and such a size, and the synodic anomaly of such and such (for these representations have been employed by almost all those who tried to exhibit the uniform circular motion by means of the so-called 'Aeon-tables',[12] but their attempts were faulty and at the same time lacked proofs: some of them did not achieve their object at all, the others only to a limited extent); but, [we may presume], he reckoned that one who has reached such a pitch of accuracy and love of truth throughout the mathematical sciences will not be content to stop at the above point, like the others who did not care [about the imperfections]; rather, that anyone who was to convince himself and his future audience must demonstrate the size and the period of each of the two anomalies by means of well-attested phenomena which everyone agrees on, must then combine both anomalies, and discover the position and order of the circles by which they are brought about, and the type of their motion; and finally must make practically all the phenomena fit the particular character of the arrangement of circles in his hypothesis. And this, I suspect, appeared difficult even to him.

The point of the above remarks was not to boast [of our own achievement]. Rather, if we are at any point compelled by the nature of our subject to use a procedure not in strict accordance with theory (for instance, when we carry out proofs using without further qualification the circles[13] described in the
H212 planetary spheres by the movement [of the body, i.e.] assuming that these circles lie in the plane of the ecliptic,[14] to simplify the course of the proof); or [if we are compelled] to make some basic assumptions which we arrived at not from some readily apparent principle, but from a long period of trial and application,[15] or to assume a type of motion or inclination of the circles which is not the same and unchanged for all planets;[16] we may [be allowed to] accede [to this compulsion], since we know that this kind of inexact procedure will not affect the end desired, provided that it is not going to result in any noticeable error; and we know too that assumptions made without proof, provided only that they are found to be in agreement with the phenomena, could not have been found without some careful methodological procedure, even if it is difficult

[12] διὰ τῆς καλουμένης αἰωνίου κανονοποιίας. In my opinion, Ptolemy is referring to a type of work in which the mean motions of the planets were represented by integer numbers of revolutions in some huge period, in which they all return to the beginning of the zodiac, and the planetary equations were calculated by a combination of epicycles or of eccentre and epicycle which was not reducible to a geometrically consistent kinematic model, i.e. to a class of Greek works which were the ancestors of the Indian siddhāntas. In this I am in agreement with van der Waerden, 'Ewige Tafeln', except that I believe that the αἰών implied by the title of these tables does not mean 'eternity' (cf. the conventional translation, 'Eternal Tables', which is philologically possible, but not necessary), but refers to the immense common period in which the planets return (cf. the Greek inscription of Keskinto, *HAMA* 698–705, and the Indian Mahāyuga). The other two references to these tables in antiquity (P. Lond. 130, see Neugebauer-van Hoesen, *Greek Horoscopes* p. 21, I 12–13, and Vettius Valens VI 1, ed. Kroll 243,8) are consistent with, but do not require, this interpretation.

[13] Literally 'as if the circles were bare [circles]'.

[14] Ptolemy in fact carries out all the proofs involving the longitudinal motions of the planets (in Bks. IX–XII) as if the motions lay in the plane of the ecliptic.

[15] The paradigm case of this is the introduction of the equant.

[16] E.g. the special model for the longitudinal motions of Mercury, or the special inclinations attributed to the inner planets for their latitudinal motions.

to explain how one came to conceive them (for, in general, the cause of first principles is, by nature, either non-existent or hard to describe); we know, finally, that some variety in the type of hypotheses associated with the circles [of the planets] cannot plausibly be considered strange or contrary to reason (especially since the phenomena exhibited by the actual planets are not alike [for all]); for, when uniform circular motion is preserved for all without exception, the individual phenomena are demonstrated in accordance with a principle which is more basic and more generally applicable than that of similarity of the hypotheses [for all planets].

The observations which we use for the various demonstrations are those H213
which are most likely to be reliable, namely [1] those in which there is observed actual contact or very close approach to a star or the moon, and especially [2] those made by means of the astrolabe instruments. [In these] the observer's line of vision is directed, as it were, by means of the sighting-holes on opposite sides of the rings, thus observing equal distances as equal arcs in all directions, and can accurately determine the position of the planet in question in latitude and longitude with respect to the ecliptic, by moving the ecliptic ring on the astrolabe, and the diametrically opposite sighting-holes on the rings[17] through the poles of the ecliptic, into alignment with the object observed.

3. {*On the periodic returns of the five planets*}[18]

Now that we have completed the above discussion, we will first set out, for each of the 5 planets, the smallest period in which it makes an approximate return in both anomalies, as computed by Hipparchus.[19] These [periods] have been corrected by us, on the basis of the comparison of their positions which became possible after we had demonstrated their anomalies, as we shall explain at that point.[20] However, we anticipate and put them here, so as to have the individual mean motions in longitude and anomaly set out in a convenient form for the H214
calculations of the anomalies. But it would in fact make no noticeable difference in those calculations[21] even if one used more roughly computed mean positions.

[17] It is not clear why the plural ('rings') is used (contrast the singular at V 1, H354,13). Although the sights are attached only to ring 1 in Fig. F (p. 218). Ptolemy is presumably referring to both ring I and ring 2, since ring 2 has first to be moved to the correct sighting position on the ecliptic ring (no. 3).

[18] See *HAMA* 150–2. Pedersen (270) has fallen into some confusion about Ptolemy's procedure: see Toomer[3] 144–5.

[19] If Ptolemy means, as we may presume, that the periods 'computed by Hipparchus' are the relationships in integers, '57 returns in anomaly correspond to 59 years and 2 revolutions in longitude', etc., then he seems ignorant of the fact that these are well-known (to us) Babylonian period relationships (for details see *HAMA* 151).

[20] This is a reference to the chapters on the 'corrections of the mean motions', IX 10, X 4, X9, XI 3 and XI7. The 'comparison' refers to the use in these chapters of *two* positions, separated by a long time-interval, to derive the mean motions. On the problem of the actual derivation of the corrections given here, and of the mean motions, see Appendix C.

[21] Ptolemy means that where he uses the mean motions in determining the eccentricity (e.g. X 7 p. 484) over the short periods involved (a few years) one could use quite crude parameters (e.g. the mean motions given by the uncorrected Babylonian periods) without seriously affecting the final result. He is right (see p. 484 n.33). The corrected mean motions are given here merely for convenience. Cf. the procedure for the lunar mean motion table, p. 179.